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COLLEGE

LABORATORY MANUAL

OF

PHYSICS

BY

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INTRODUCTION.

Most of the Exercises described in this book have been used by the writer for a long time as a laboratory accompaniment to a general course of lectures and text-book study extending through one college year. The student's formal weekly engagements in this course are two lectures and one two-hour laboratory exercise. A part of the lecture hour is often taken for an informal examination of the class by means of oral questions on matters treated in the text-book or in the laboratory.

These Exercises do not profess to give training in exact measurement. Their primary object is first-hand acquaintance with a wide range of important facts, principles, and apparatus, or machines.

In such exercises the students must often work in groups of considerable size with a single piece or set of apparatus; for example, it is impracticable to supply a steam-engine for every student or every pair of students making up a laboratory section of, let us say, sixteen members. But it should be the constant endeavor of the teacher to provide individual work and to foster individual responsibility, wherever this can be done, by reduplication of the less elaborate and expensive apparatus or by the definite division of labor in complicated operations. Thus, in Exercise 6, on *Flotation,* each student should work entirely by himself; in Exercise 16, *Horse-power of a*

Steam-engine, each student in turn should be required to take an indicator diagram and work out the horse-power.

Any one of several well-known college text-books of physics gives a suitable foundation or background for most of the Exercises here described; but it has seemed desirable to supplement in some measure the ordinary treatment of Torricelli's Law and Fluid Friction, and this has been done in the Appendix.

Mr. Arthur Kendrick of the *International Instrument Company*, Cambridge, Mass., has had better opportunities than any other manufacturer for examining those forms of apparatus which are peculiar to this course; and any teachers who may wish to procure such apparatus would do well to apply to him.

EDWIN H. HALL.

CAMBRIDGE, September, 1904.

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COLLEGE LABORATORY MANUAL.

EXERCISE 1.

BALANCES.

MECHANISM AND THEORY OF A SCALE-PAN BALANCE.

Before taking the balance apart study it carefully in its position of rest and in its position of use, and make sure of being able to replace properly each part which is to be displaced for the purposes of this Exercise.

The three wedge-shaped pieces of steel which are fixed in the *beam* of the balance are called *knife-edges*. The acting edge of the central wedge is turned downward and is the line of support of the beam and of all that the beam carries. The acting edge of each end piece is turned upward and is the line of support of the corresponding pan. The weight of everything which hangs freely from either end edge may be regarded as applied directly at that edge; therefore, if the balance is properly constructed, with the three acting edges in one plane, the parts suspended from the two end edges will balance each other in all proper positions of the beam, even when it is considerably inclined from the horizontal. Accordingly, the sensitiveness of the balance will be the same without the pans as with them; and we shall for convenience dispense with them in this Exercise.

Carefully remove the pans, including whatever detachable things are suspended from the end knife-edges, and place them in a safe position where they will not be lost or mixed with other similar articles.

Observe whether the three knife-edges are in the same plane.

Fix on the pointer the *bob* b, a thick-walled piece of brass tubing containing a cork stopper, the lower end of the *bob*

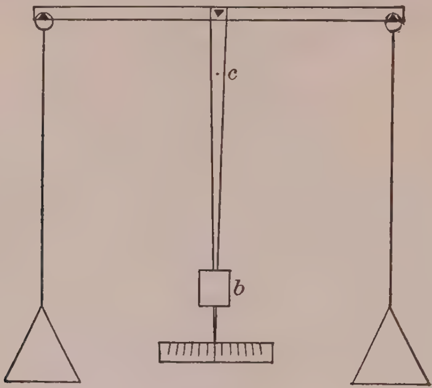


FIG. 1.

being just even with a scratch made near the lower end of the pointer. The object of this attachment is to bring the centre of gravity, c, of the beam, including the pointer and the bob, a considerable distance, perhaps 2 cm., below the central knife-edge.

Place the beam, as now loaded, in a horizontal position, with the pointer also horizontal, and balance the whole on some sharp horizontal edge. Measure the distance, parallel to the pointer, from this edge to the central knife-edge. Call this distance r.

Measure l, the distance from the central k.e. to either end k.e.

Measure l, the length of the indicator, from the central k.e. to the top of the scale when beam is in position for use.

Find s, the average length of the divisions of the scale, at the top of the lines.

Find, by use of another balance, the weight, M, of the beam, as now loaded.

Result: Find, by calculation from the data indicated above, through how many divisions of the scale the pointer. *as now loaded*, would be permanently deflected by a load of 1 gram suspended from either end k.e. Little error and much simplification will result from treating the upper edge of the scale as a straight horizontal line in this calculation.

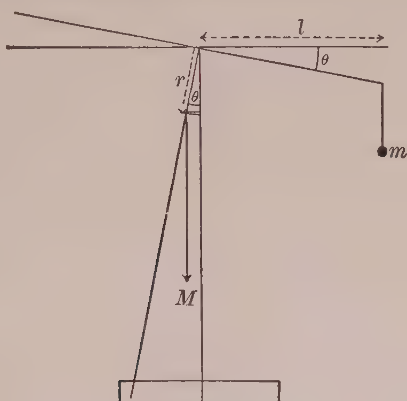


FIG. 2.

Let m in Fig. 2, represent the 1-gram load, and let θ be the permanent deflection produced by it. Then the condition for equilibrium is

$$m \times l \times \cos\theta = M \times r \times \sin\theta,$$

whence

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{ml}{Mr}.$$

Let n = number of divisions of deflection. Then

$$ns = I \tan \theta = I\frac{ml}{Mr},$$

or, since $m = 1$,

$$n = \frac{Il}{Msr}.$$

The result thus attained can be tested by replacing the pans of the balance and putting 1 gram in either pan, the bob still remaining on the pointer.

MECHANISM AND THEORY OF A PLATFORM BALANCE.

The ordinary platform balance of the laboratory has its one or two platforms raised well above all the knife-edges and some device, more or less hidden, for keeping each platform horizontal when the beam of the balance tips.

Study the balance carefully and then make a very simple sketch of its *jointed mechanism:* 1st, with the beam horizontal; 2d, with beam inclined, making points which do not move in the tipping of the beam common to both

FIG. 3.

diagrams; that is, superpose one diagram on the other. Exaggerate the tipping. Fig. 3 is a hint of the kind of sketch required.

Explain why with this balance a weight placed at the inner edge of a pan, near the central knife-edge of the beam, has the same * effect as if it were placed at the outer edge, much farther from the central fulcrum.

* Owing to imperfections of construction of the balance this is not quite true.

EXERCISE 2.

VALUE OF g BY ATTWOOD'S MACHINE.

Let M be the mass in grams of each of the two main weights in Fig. 5. Each should be about 500 grams, and the difference between the two should not be greater than 0.05 gram.

Let μ be the allowance for friction, that is, the mass which, added to either M, will just maintain *uniform* motion over the pulley, one weight going down and the other going up. This is to be found by trial.

Let m' be the mass which we are to add to the right-hand M, M_1, to make it go down with increasing velocity. It may well be 10 grams.

Let m be the *virtual* mass of the pulleys in the case of increasing velocity, that is, the mass which, if concentrated in the rim of the uppermost pulley, and therefore undergoing the same changes of velocity as the thread and the attached weights, would resist the acceleration just as much as the pulleys resist it. The manner of finding m by experiment will be given below.

With m' placed on M_1 in Fig. 5 and with the supporting platform of M_1 removed, we have as the net accelerating force $(m' - \mu)g$ dynes, and as the mass to be set in motion $(2M + m' + m)$ grams, practically. This gives us, if α is the rate of acceleration,

$$(m' - \mu)g = (2M + m' + m)\alpha,$$

or

$$g = \frac{2M + m' + m}{m' - \mu} \alpha. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

We have, therefore, only to find the various quantities on the right-hand side of the equation in order to find g. M and m' are already known or can be found by simple weighing; μ, m, and α are not quite so easy to deal with.

To find μ, add small weights to M_1, Fig. 5, until it has such a load that, once started with a small velocity downward, it will continue to move without change of this velocity. The load is then μ.

Take care to have the bearing surfaces of the pulleys free from dirt and lightly oiled before making this test.

To find m we make use of a "torsion balance", an instrument shown in Fig. 4. Remove the top pulley, P, of the Attwood's machine and fix this pulley at the bottom of the torsion balance. Turn D, the disk of the torsion balance, half-way round its vertical axis, thus twisting the suspending wire, and then set it free. Note carefully the time required for 100 single oscillations. Then remove P and, after noting that the time of oscillation is now considerably less than before, put in place in a groove on the fixed disk, D, of the torsion balance a ring, R, indicated by the dotted lines. This ring, which accompanies the apparatus, is made with a mean radius equal to the radius of the groove in P, and is of such a weight that when it replaces P on the torsion balance, the time of oscillation of the balance is the same as when P was

FIG. 4.

carried, as observation will show. Accordingly (see definition of m), the weight of this ring is that part of m which is due to the main pulley, P, of the Attwood's machine. If the four supplementary pulleys are of about the same size and weight as P, the allowance to be made for each one of them bears to the allowance to be made for P the ratio which its velocity of revolution bears to that of P. This ratio is $r \div R$, where R is the radius of P and r is the radius of that part of its axle which rests on the other pulleys. Accordingly, if w is the mass of the *ring*, we have

$$m = w + 4w \times \frac{r}{R}.$$

To find α, get the distance, s cm., which M_1 descends in t seconds, and then use the formula

$$s = \tfrac{1}{2}\alpha t^2. \qquad (2)$$

The measurement of s and t requires considerable care, as the following directions will show:

Adjust the torsion pendulum T, Fig. 5, so that it will, when released after being turned once completely around, make at least thirty single swings before coming to rest, and so that these thirty swings shall take just thirty seconds. The first of these two conditions is attained by so adjusting the two controlling brackets, b and b, which keep the rod from swaying, that friction between them and the rod shall be small; the second, by moving two heavy nuts, N and N, which are carried on a horizontal bar near the bottom of the rod, toward or from each other, thus diminishing or increasing the time of a single swing.

See that the silk thread which is to suspend the weights is in good condition and of the right length to let one

weight clear the floor by 15 or 20 cm., when the other rests on the platform at the top, as in Fig. 5.

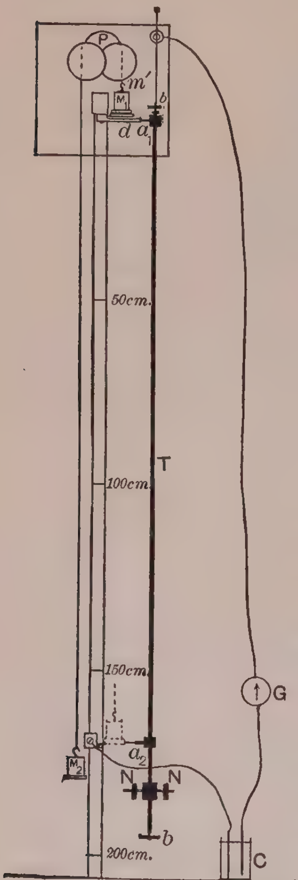


FIG. 5.

Place the small weight m' on the top of M_1 resting on the platform, and arrange the swinging bar, d, which keeps the platform in place, so that a very slight movement of d will drop the platform and release M_1.

Adjust the horizontal arm a_1, which is carried near the top of the pendulum-rod T, so that it will, when the pendulum is in its natural position of rest, just touch the bar d at a point where a slight push would swing d clear of the platform.

Adjust the lower arm, a_2, so that it will, when T is at rest, point toward the suspending silk thread, and make the fine copper wire, which projects from the end of a_2, so long that, in its present position, it would touch the bottom flange of the descending M_1, but not so long that it could reach the main part of M_1. Arrange a similar wire, as in Fig. 5, to touch on the opposite side of the flange of the descending M_1, and make connections with a galvanic cell, C, and with a

portable astatic galvanometer,* G, according to the indications of Fig. 5.

Everything being in readiness, turn T half-way around, then release it. Passing through its position of rest it frees M_1. If in its descent M_1 closes the electrical circuit, so that the galvanometer is affected, we know that it reaches a_2 in some whole number of seconds from the time of its start. If the electrical contact is not made, it will be plain that a_2 is too high for an n-seconds fall and too low for an $(n-1)$-seconds fall. It must then be raised or lowered till the descending M_1 makes contact; and when this occurs s and t are readily found, and so α is known.

As the two M's may differ a little, it is well to repeat the determination of μ and of α with M_2 on the right-hand side and M_1 on the left-hand.

* It is quite practicable to dispense with all these electrical contrivances and depend on the eye alone to tell whether the flange of M_1 is opposite a_2 when the pendulum is at midstroke.

EXERCISE 3.

PENDULUMS.

Value of g by Simple Pendulum.

The "simple pendulum" as here used is a sphere of lead, about 1 cm. in diameter, suspended by a fine silk thread from the beveled edge of a wooden shelf, S, Fig. 6. The length, l, of the pendulum may, with sufficient accuracy for our present purpose, be measured from the lower edge of the shelf to the centre of the ball. It is well to have l as great as 100 cm. The width of the swing should be small, perhaps 5 cm. each way from the middle.

Fig. 6.

Take the time required for 100 single swings, and so find t, the time of one swing. Then from the formula

$$t = \pi\sqrt{\frac{l}{g}}$$

the value of g is readily found.

Properties of Reversible Pendulum.

1°. In Fig 7 and Fig. 8 R is a meter-rod having a short cylindrical cross-bar, b_1, fixed permanently near one end, and a similar cross-bar, b_2, adjustable at any position along a slot of considerable length in the lower half of the rod.

Near the lower end of the rod there may be a metal bob, B. If the slot in R is properly placed, it is possible to fix b_2 in such a position that R will vibrate in precisely the same time when suspended, inverted, from b_2, as when suspended in the position shown by Fig. 7. Accordingly R, with its attachments, is called a *reversible* pendulum.

To show this reversibility, place b_2 at the middle of the slot and hang alongside, as in Fig. 7, a simple pendulum, having the centre of the ball just on a level with the upper side of b_2. The *length* of the simple pendulum, as previously defined, is now equal to the distance between the under side of b_1 and the upper side of b_2. Set the two pendulums vibrating together. If the simple pendulum is the slower, shorten it, and move b_2 up accordingly; and *vice versa*. Continue the trials until the time of vibration of the two pendulums is the same. It will then be found that the time of vibration of R suspended from b_2 is also equal to the time of vibration of the simple pendulum.

FIG. 7.

FIG. 8.

2°. In Fig. 8 is shown a pin, p, extending upward from the bracket which carries R. Behind p is a similar pin, not shown, of equal height.

Suspend R, by its cross-bar b_1, from the top of these two pins. Then by means of a hammer, h, Fig. 9, carried by a spring, S, mounted on some convenient base, strike R a sharp blow just above b_2 and note the direction in

which b_1 leaves p. Then in the same way, after replacing the pendulum, strike with h a little below b_2, and note the direction in which b_1 now leaves p. The results will probably point to the conclusion that, if we could tap R exactly on a level with the upper side of b_2, it would not be dislodged from p, the blow producing no movement of the line of contact between b_1 and p, but merely rotation about this line as an axis. That is, the upper side of b_2 is the "centre of percussion" for R when the under side of b_1 is the point of suspension.

FIG. 9.

EXERCISE 4.

THE BALLISTIC PENDULUM; VELOCITY OF RIFLE BULLET.

The main mass of the ballistic pendulum, P in Figs. 10 and 11, is a strong tubular cylinder of brass containing a thick plug of soft wood, which plug is exposed at one end, in order to receive the bullet, but is backed at the other end by a strong plate of brass so held that it cannot be driven out by the impact of the bullet.

This pendulum is suspended by four wires from a beam at the top of the room. Fig. 10 gives an end view of P. Fig. 11 gives a side view of P and shows also in position the rifle R, mounted so that its line of sight is horizontal, a frame F holding a paper diaphragm through which the bullet is to be shot, the object of the paper being to shield P from the blast of air from the rifle, and finally the scale S, along which the pointer at the bottom of P moves after the bullet strikes.

Fig. 10.

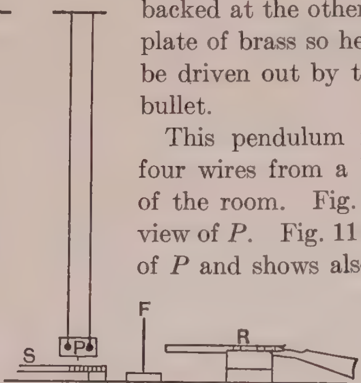


Fig. 11.

The suspending wires are attached to P by means of binding-posts, which make it possible to put the axis of P horizontal and on a level with the axis of the rifle-bore.

Adjustments having been made according to the indications given above, fire a bullet into P and note the distance P moves from rest at its first swing. Call this distance D. Repeat the firing two or three times and take the mean value of the several D's noted. Measure the distance from the middle of P to the level of the top of the suspending wires, and call this distance L.

Detach P and find by weighing its mass in grams, M.

Take a bullet from a cartridge and find its mass to 0.05 gram; call this m.

From L and D find h, the distance the centre of gravity of P *rose* in the swing caused by the impact of the bullet.

From h find V, the velocity given to P by the bullet. (Since all the kinetic energy involved in the velocity V is spent in *raising* P, and the lodged bullet, during the swing D, the relation of V to h is the same as if P moved straight up the distance h.)

From M, m, and V find v, the velocity of the bullet just before it struck P. (In this calculation the fact that P gains in mass by taking in the bullet may be ignored, as the mass of the bullet is very small compared with that of P.)

EXERCISE 5.

VOLUME ELASTICITY OF WATER.

The water to be compressed is contained in a bulb, B, Fig. 12, and its graduated stem, S. The process of filling the bulb has been such as to drive the air from the water by continued boiling. The weight of the empty bulb and stem is known, and so the weight of water within it can readily be found. From this weight, and the temperature, the volume of the water before compression can be found.

The bulb, fastened on a thin plate of metal, is placed upright in a glass jar, J, which is filled with water to the bottom of the piston P, which at first is some little distance above the inner end of the stop-cock O. Pressure is applied by forcing down P, by means of a screw not shown in Fig. 12. The increase of pressure is measured by means of the graduated pressure-gauge G, which contains air. The pressure on the water in J is transmitted to the interior of B through the mouthpiece M, Fig. 13, which contains air

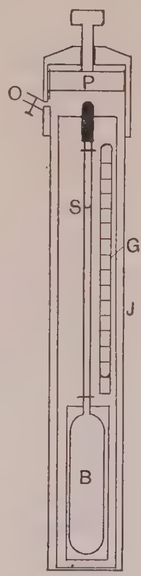


FIG. 12.

FIG. 13.

and is of considerable length in order to prevent the outer water from going over into the stem S and joining the column there. The change of volume of the water in B, when P is forced down, is estimated from the fall of the top of the column in S. The size of the divisions of S can be found from certain observations made before the water was put into B. The increase of pressure diminishes slightly the dimensions of B and its stem in every direction and therefore makes the interior space a little smaller, a fact which tends to keep the water from descending in the stem. Multiplying the observed descent by 1.06 will give, with sufficient accuracy for the present purpose, what the descent would have been if the bulb had suffered no change from the increase of pressure.

Take a reading of the bottom of the water meniscus in G, with O open and piston well up; this reading indicates the volume of the air in G, and is to be called A_1. Under the same circumstances take a reading, as accurate as practicable, of the water in the stem, and call this h_1.

Then close O and force down the piston P till the air column in G is reduced to about one sixth of the original volume; read the reduced volume and call it A_2. Keeping this air volume unchanged, take a new very careful reading of the water in S, and call it h_2.

Raise the piston till the air column in G is nearly A_1; then open O, thus restoring the original pressure. Read height of water once more in S and call this last reading h_3. If h_3 is not just the same as h_1, take $\frac{1}{2}(h_1 + h_3)$ as the reading at initial pressure.

Repeat all these observations a number of times and take means.

Read the barometer and record its height as H.

V, the volume in cu. cm. of water before compression,

is supposed known. The capacity, c, of a single division of S may be given directly or may be calculated from the known number of grams of mercury, density 13.6, required to fill a certain number of these divisions.

Pressure is to be reckoned in *dynes* per sq. cm. It is to be noted that the pressure on the air in G at the beginning was somewhat greater than the atmospheric pressure, because the bottom of the air column was considerably below O. If this difference of level is n cm., we have for the initial pressure $(H \times 13.6 + n)g$ dynes per sq. cm. From this value and from the readings A_1 and A_2 the greatest pressure, and so the increase of pressure, can be found.

From the increase of pressure, the decrease in volume of the water and the original volume of the water the coefficient of volume elasticity of water is to be calculated.

EXERCISE 6.

FLOTATION: CENTRE OF GRAVITY AND CENTRE OF BUOYANCY.

In this Exercise we make such measurements and weighings as will enable us to find the position of the centre of gravity of the whole floating body and the centre of buoyancy of the whole floating body in two conditions:

1st, when the body floats upright in equilibrium;

2d, " " " " inclined " " .

We then look to see whether the second position of the c. of b., as found, lies on the vertical line through the second position of the c. of g., as found.

The floating body, see Fig. 14, consists of a paraffine-soaked block of wood, *B*, having its length somewhat greater than its width and its width greater than its thickness, from which rises a mast *m*, carrying a slider *S*, which can be moved up or down in order to give the right degree of top-heaviness to the whole, a triangle *t*, on which is an arc marked off in degrees, and a small plummet, *p*, hanging from a pin at the upper corner of the triangle, which point is the centre of curvature of the arc shown on *t*. Resting on *B* is a metal plate *d*, in which is a slot through which the mast extends. The object of *d*, which has a good deal of freedom of motion on *B*, is to balance, or

"trim", the whole so that it may float upright or float with the same amount of inclination in either of two opposite directions.

The weight of the plummet is practically applied at its point of suspension, and M_p stands for the weight of the plummet and of the pin from which it is suspended, if there is need of taking account of this pin. The values of M_p, of M_t, weight of t, of M_m, weight of mast (above B), of M_s, weight of S, and of M_d, weight of d, are all to be furnished to the student, as he cannot find them for himself without taking the apparatus apart. The value of M_B, weight of the block (and of the imbedded part of the mast, which will be regarded as part of B), is to be found by weighing the whole and subtracting from M, the total weight, the weight of the several parts above mentioned.

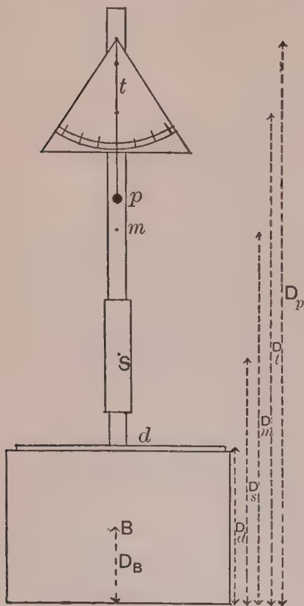


Fig. 14.

After the weights are all known adjust S and d so that the block will float in equilibrium when tipped about 20° either way, parallel to the plane of t, from its upright position, if this can be done without bringing either edge of the upper surface down to the water; but after this adjustment make no change in the position of S or of d. Note the amount of inclination of the block in each direction,

as above mentioned, and record the mean inclination in degrees, calling this θ.

Then measure the various D's, D_B, D_d, etc., indicated in Fig. 14.

D_B is distance from bottom of D to c. of g. of D;
D_d is distance from bottom of D to c. of g. of d;
etc.

Measure L, the length of the block.
" W, the width of the block.

Draw on sq. mm. coordinate paper, on a scale of 4 to 1, a rectangle, $ABCD$, Fig. 15, in which AB represents the

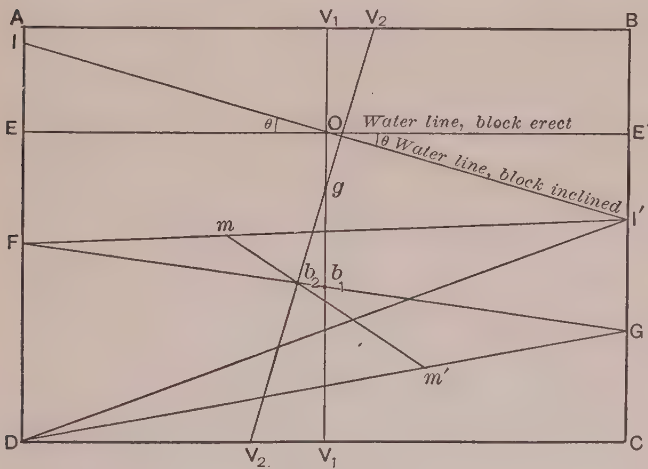


FIG. 15.

width and AD the thickness of the block, and consider this rectangle as a section through the centre of the block.

Draw the line V_1V_1 parallel to and midway between AD and BC.

From the total weight M and the dimensions L and W

calculate the depth to which the block is sunk when floating upright, and mark accordingly EE' on Fig. 15.

From the various weights and dimensions indicated in Fig. 14 calculate the distance of the c. of g. of the whole from the bottom of the block, and mark by a point, g, on the line V_1V_1, the position of this c. of g. in Fig. 15.

Mark the point b_1, half-way down from O, the crossing of V_1V_1 and EE', to the bottom, as the c. of b. for the upright position of the float.

To find the position in which II' must be drawn, observe that the trapezoid $II'CD$, representing the submerged part of the block inclined, must have the same area as the rectangle $EE'CD$. Therefore the triangle IOE must equal the triangle $I'OE'$, and accordingly O must be the centre of both water-lines. The distance EI must equal $EO \times \tan\theta$. EI being thus found, the point I is marked, and then a straight line is drawn through I and O. This line is II'.

The c. of b. for the inclined position is the c. of g. of the trapezoid $II'CD$. To find this point proceed as follows: Draw the median line FG from the middle point of ID to the middle point of $I'C$. The required point is somewhere on this line. Draw the line DI', dividing the trapezoid into the triangles $II'D$ and $I'DC$. Mark the c. of g. of the larger triangle, a point m one third of the distance along the straight line from F to I'; mark the c. of g. of the smaller triangle, a point m' one third of the distance along the straight line from G to D. The c. of g. of the trapezoid lies on the straight line connecting m and m', and as it lies also on the line FG, it must be at the point of crossing, b_2. This is the new c. of b.

Through b_2 draw the straight line V_2V_2 at right angles with the line II'. V_2V_2 is the vertical through the c. of b. when II' is the water-line, and therefore V_2V_2 should be found to pass through the c. of g. of the float, if everything in this Exercise has been correctly done.

EXERCISE 7.

SURFACE TENSION OF WATER AND OF ALCOHOL.

Take a piece of thermometer tubing about 8 cm. long and 0.1 cm. or less in diameter of bore and, after washing it out thoroughly with a soap solution, rinse it thoroughly inside and out by holding it for one minute vertical in a stream of clean water escaping from a tap.* It is hardly worth while to use distilled water in a hasty experiment. After the rinsing, and later whenever there is need, remove any drop of water which may be sufficient to bridge the bore by giving the tube a sharp flirt, never by blowing the breath through it, for this latter operation is almost sure to foul the tube by introducing saliva.

Fill some vertical-walled clean glass vessel with clean water to a depth of 5 cm. or more. Holding the tube by its top, push its lower end some centimeters deep into this water, then slowly raise it, noting that the capillary column within it increases in height at first as the tube is lifted but presently reaches a maximum. Then measure this height, h, Fig. 16, from the level surface of the water in the vessel to the middle of the concave top of the column, keeping the measuring-stick outside the vessel. There must be no break in the column or bridge of liquid

* If several tubes are to be cleaned at once, let them stand vertical for some minutes in a tumbler filled with soap solution; then stand them for a minute or two vertical in a tumbler receiving a vigorous stream from a tap.

across the bore above the column when this measurement is made. Call the height of the water column h_w.

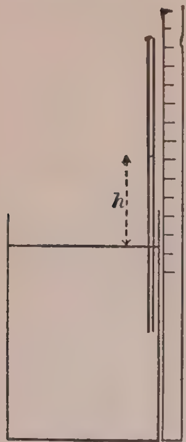


FIG. 16.

Rinse out the water from the tube by running two or three columns of alcohol through it, and then with a vessel of alcohol repeat the operation and measurement just made with water. Call the height of the column h_a.

To find r, the radius of bore of the tube, proceed as follows: Dry the tube by gentle heating, push it vertically into a mercury well till it is nearly submerged; close the top by pressure of thumb or finger; lift it gently thus closed and lay it horizontal; measure the length of the mercury column within it; pour this mercury into a wooden or paper box the weight of which is known to 0.01 gram; weigh the box and its contents to the nearest 0.01 gram and find the weight of the mercury; find its volume in cu. cm. from its weight and its density, 13.6; from its volume and the length in cm. it had in the tube find the area of cross-section of the column in sq. cm.; and from this find the radius r of the bore of the tube in cm.

In using the formula

$$T = \frac{hr\rho g}{2 \cos \alpha},$$

for finding T, the surface tension in dynes per centimeter, ρ, the density of the column, is to be taken as 1 for water and as 0.8 for alcohol; and α may, with a pretty close approach to accuracy, be taken as 0° in both cases.

EXERCISE 8.

FLOW OF WATER.

Water enters a stand-pipe P, Fig. 17, keeping it full to overflowing, while a horizontal stream escapes from an orifice O, which may have any one of several different

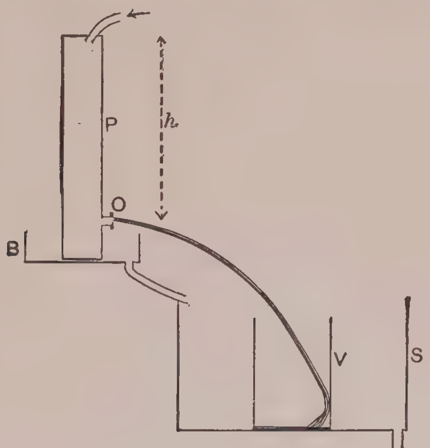


FIG. 17.

shapes, into a vessel V of known capacity standing in a sink S. The overflow from P falls into the basin B, whence it runs off into S. The diameter of P is about 10 cm., its total height about 60 cm., of which some

10 cm. is below O. The capacity of V should be as much as 3 liters.

The various orifices used are screwed on in turn to the short side tube of P. The description of these orifices follows, the dimensions being given approximately:

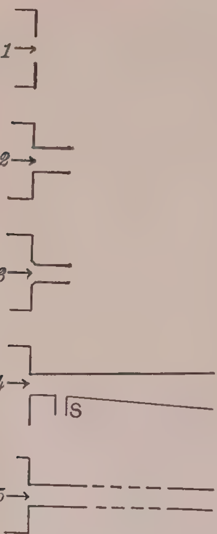


Fig. 18.

	Length.	Diam. of Inner End.	Diam. of Outer End.	Area of Outer End in Sq. Cm.	Time, in Seconds, to Fill V.
	cm.	cm.	cm.		
No. 1→	0.0	0.6	0.6	......	......
No. 2→	1.0	0.6	0.6		......
No. 3→	1.0	0.6	0.5	......	......
No. 4→	5.0	0.6	1.0		.... (1) .... (2)
No. 5→	50	0.6	0.6	......	......

The last column in the preceding table is to be filled from the observations made by the student. (1), against No. 4, is to be taken with the side tube S open, and (2), against the same orifice, with S closed.

Try also the experiment of connecting S, by means of a

short rubber tube, with water in a small vessel beneath, the stream meanwhile running through the flaring outlet.

Try the like experiment with a tube of the same bore as the inner end of No. 5 thrust into the flaring part, so as to make the channel of the same cross-section all the way. See Fig. 19.

FIG. 19.

No. 5, which is intended to illustrate the effect of friction in opposing flow, has a number of small holes, not shown in Fig. 18, at equal intervals along its upper side. These holes are, during the main use of No. 5, closed by rubber rings; but when these rings are pushed aside the different heights of the small upward jets of water indicate the fall of pressure from the inner to the outer end of the tube.

Finally, measure the height h, Fig. 17.

ILLUSTRATION OF TORRICELLI'S LAW.

This illustration is made with the data from orifice No. 3, which is long enough to deliver a cylindrical stream, not a tapering stream such as issues from No. 1, but is not long enough to give much effect to friction.

From the known capacity of V and the number of seconds required to fill it from No. 3 find the number of cu. cm. delivered by this orifice per second. From this volume per second and from the area of cross-section of the issuing stream (outer end of No. 3) find the velocity of issue. Call this v_o, the observed velocity.

From the height h, Fig. 17, find by Torricelli's law,* $v=\sqrt{2gh}$, what the theoretical velocity is, and call this v_t.

Compare v_o and v_t.

* The derivation of this law will be given in the Appendix.

Estimation of Cross-section of Vena Contracta.

Vena contracta, or contracted vein, is the name given to the narrowest part of the stream issuing from a simple hole in a thin plate, like orifice No. 1. This part is some little distance outside the hole in the plate.

Find the number of cu. cm. delivered per second into V from No. 1, and, assuming the velocity of flow at the vena contracta to be that given by Torricelli's law, as already calculated in the case of orifice No. 3, find the area of cross-section of the stream at the vena contracta. Call this A_c. Find what per cent A_c is of the area of the hole through which the stream issues.

Consideration of Friction.

Find the number of cu. cm. delivered per second into V from No. 5, and then, assuming that the issuing stream has the same area of cross-section as the bore of the tube, find the velocity of issue. Call this v_o.

The theoretical velocity in a case like this, where friction is to be taken into account, is found from the formula *

$$v = \sqrt{2gh \div \left(1 + k\frac{l}{d}\right)},$$

where g and h have the same meaning as in the simple Torricelli formula, l is the length of the tube, d is the diameter of bore, and k is a quantity which depends on the condition of the interior of the tube. In this Exercise 0.03 may be taken as the value of k. Call the value of v thus obtained v_t.

Compare, for No. 5, v_o with v_t.

* The derivation of this formula will be given in the Appendix.

EXERCISE 9.

FLOW OF AIR.

Exercise 9 undertakes to make with air experiments similar to a number of those made in Exercise 8 with water. The general formulas for the flow of air, including the case where friction has to be considered, are very similar to the corresponding formulas for the flow of water, but a marked and troublesome difference arises from the fact that the density of any gas is much affected by change of pressure and of temperature. If, however, we make the changes of pressure small compared with the original pressure, and eliminate or keep within narrow bounds the temperature changes, we may, in the cursory treatment of the matter which we here undertake, ignore in some respects the change of density which the air undergoes in its flow.

T in Fig. 20 is a copper tank, such as is used with a kitchen range as a hot-water reservoir, holding perhaps 30 gallons. Its capacity expressed in cu. cm. is supposed known within two or three per cent.

Into *T*, by way of the aperture *A*, air is pumped. The pressure-gauge *G* contains water, so that the excess of height of the liquid in the outer arm over that in the inner arm gives directly, in grams per sq. cm., the

excess of pressure in T above the atmospheric pressure. The latter pressure also is to be expressed in grams per sq. cm., and, assuming the normal barometric height, we may take it as 76×13.6, or 1033. If the air is forced into T so fast as to raise the pressure therein rapidly, the compression of the air raises its temperature enough to increase the pressure perceptibly, so that when the inflow has ceased and the temperature within T begins to fall the gauge G indicates a slow fall of pressure, as if air were leaking out from T. If there is no real leak, the gauge will after a few minutes cease to show a change. When this condition is reached, the gauge should be read carefully, and, if the height of the water in the outer arm above that in the other arm is not more than 30.2 cm. or less than 29.8 cm., this difference of height should be recorded under the heading Δ_b in the table below, the subscript b indicating that this difference of pressure exists *before* outflow of air. If the reading of G lies outside the limits mentioned above, air should be cautiously forced in or let out till the proper value for Δ_b is obtained.

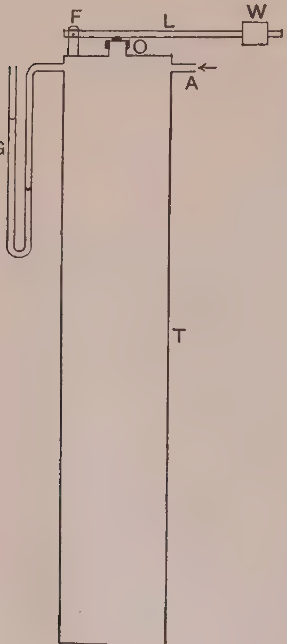
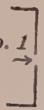
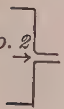
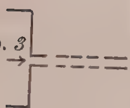


Fig. 20.

Outflow occurs, when it is wanted, through an opening in the cap O. The opening in the cap may answer to any one of the descriptions following.

	Length in Cm.	Diam. in Cm.	Δb, Gm. per Sq. Cm.	Mean Δb.	Δc, Gm per Sq. Cm.	Mean Δc.	Δa, Gm. per Sq. Cm.	Mean Δa.	t in Seconds.
No. 1 	0.0	0.226	....	...	....	...	....	...	....
No. 2 	0.5	0.193	....	...	....	...	....	...	....
No. 3 	50	0.226	....	...	....	...	....	...	....
FIG. 21.									

From No. 1 and No. 2 outflow is started by suddenly and fully raising the lever L, pivoted at F and carrying the weight W, which holds a soft-rubber pad down on the opening in the cap O. It is stopped by suddenly and firmly putting L down. A little elevation of L permits outflow, but not unobstructed outflow. Careless application of the stopper may not entirely prevent outflow. With No. 1 or No. 2 flow should be permitted for 5 seconds. Much care should be taken in the measurement of this interval of time. It is well for the person who is to work the lever L to have in plain view a pendulum beating seconds and to practice starting and stopping the outflow at the instant when the pendulum is passing through its middle position.

With No. 3 the lever stopper cannot well be operated

and a finger or thumb should be used instead. Flow from this orifice should be permitted for 10 seconds.

The outflow from any orifice is of course accompanied by some expansion of the air in the tank and this expansion causes a temporary fall of temperature. After the flow has stopped the air remaining in the tank takes up heat from the metal wall surrounding it and presently, after perhaps one or two minutes, has practically its original temperature. During this rise of temperature the pressure also rises. A reading of the pressure-gauge should be taken immediately after the outflow ceases, and should be recorded as Δ_c. Another reading should be made after the temperature and pressure have become stationary, and this should be recorded as Δ_a.

The average internal excess of pressure during the flow, to which excess of pressure the flow is due, is approximately $\frac{1}{2}(\Delta_b+\Delta_c)$. The amount of air remaining in the tank after the outflow is $\frac{1033+\Delta_a}{1033+\Delta_b}$ of the amount which was there before the outflow. If the capacity of the tank is C cu. cm., the amount of air which has flowed out would occupy, at the pressure existing in the tank before outflow, $\frac{\Delta_b-\Delta_a}{1033+\Delta_b}C$ cu. cm., and, although in flowing out it has expanded somewhat, there will be no great error in taking the value just given as the volume of air which passed through the orifice in t seconds. The density of this air at temperatures near 20° C. may be taken as 0.00124.

Illustration of Torricelli's Law.

Find the number of cu. cm. of air delivered per second by No. 2, and then, assuming the cross-section of the stream to have been the same as the cross-section of the

bore of the tube at the outer end, find the velocity of outflow needed to give this delivery. Call this velocity v_o, the observed velocity.

From Torricelli's law for a gas,

$$v=\sqrt{2g\times\frac{\frac{1}{2}(\Delta_b+\Delta_c)}{0.00124}},$$

find what the theoretical velocity is for the same case, and call this v_t.

Compare v_o and v_t.

ESTIMATION OF CROSS-SECTION OF VENA CONTRACTA.

Find the number of cu. cm. of air delivered per second by No. 1, and then, assuming the velocity of outflow to have been that given by Torricelli's law (with the value of $\frac{1}{2}(\Delta_b+\Delta_c)$ which belongs to No. 1) find the area of cross-section of a cylindrical stream which, with this velocity, would give the actual delivery. Call this A_c.

Find what per cent A_c is of the area of the hole in No. 1.

CONSIDERATION OF FRICTION.

Find the number of cu. cm. of air delivered per second by No. 3, and then, assuming the issuing stream to have been the same as the cross-section of the bore of the tube, find the velocity of outflow needed to give this delivery. Call this v_o, the observed velocity.

Find the theoretical velocity from the formula

$$v=\sqrt{2g\times\frac{\frac{1}{2}(\Delta_b+\Delta_c)}{0.00124}\div\left(1+k\frac{l}{d}\right)},$$

where l is the length of the tube, d is the diameter of the bore, and k a quantity which we may take here, as in Exercise 8, to have the value 0.03. Call the value of v thus obtained v_t.

Compare, for No. 3, v_o and v_t.

EXERCISE 10.

EXPANSION OF MERCURY.

The method here followed is called the method of balanced columns.

Fig. 22 gives a general idea of the apparatus used. The mercury is contained in a glass U-tube having a stop-cock C at the bottom. The left-hand branch of this tube, as here shown, is inclosed in a brass tube B_c, through which cold water flows. The right-hand branch is inclosed in a similar, but slightly longer, brass tube B_s, through which steam flows. The upper parts of the glass tube are made large in order to avoid difficulty with capillary action.

Let t_s be the temperature, centigrade, of the stream of cold water.

Let t_c be the temperature of the steam, to be found from the barometric pressure by the use of a "steam table," that is, a table of steam data.

Let L_c be the length, in cm.

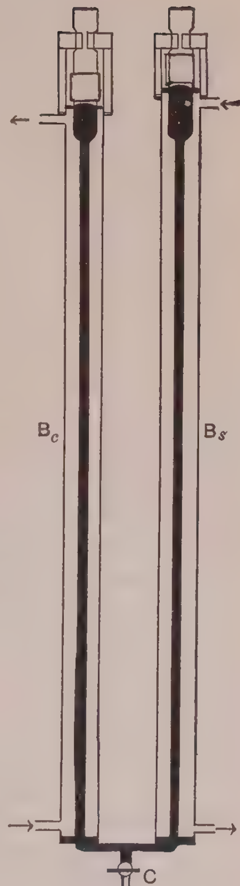


FIG. 22.

the cold column, when the apparatus is in action and the columns are in equilibrium.

Let l be the excess of height of the hot column. This can be measured by means of a cathetometer, or by means of a device indicated by Fig. 23, where S_1 and S_2 are close

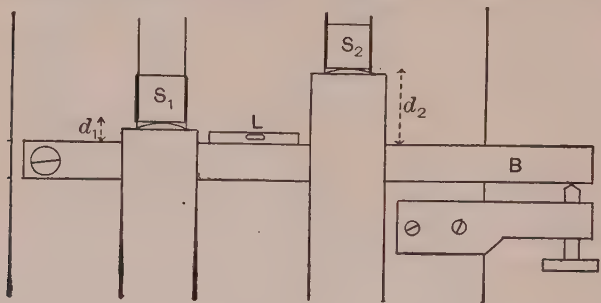


FIG. 23.

fitting sleeves of brass and B is a straight-topped bar, pivoted at one end, supported by a leveling screw at the other end, and carrying a small spirit level, L. The lower end of each sleeve being put just on a level with the top of the adjacent mercury meniscus and B having been leveled, the distances d_1 and d_2, respectively, from B to the bottom of S_1 and the bottom of S_2, are taken, and the difference of these distances is l. It should be measured very carefully.

Let ρ_c stand for the density of the mercury at temperature t_c and $v_c\left(=\frac{1}{\rho_c}\right)$ stand for the volume of 1 gram of mercury at this temperature. Let ρ_s and v_s have corresponding meanings for the heated mercury.

When there is equilibrium, we have

$$L_c\rho_c=(L_c+l)\rho_s, \quad \text{or} \quad \frac{L_c}{v_c}=\frac{L_c+l}{v_s},$$

or

$$\frac{L_c+l}{L_c}=\frac{v_s}{v_c}, \quad \text{or} \quad \frac{l}{L_c}=\frac{v_s-v_c}{v_c}. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

But by very definition of K, the coefficient of expansion, we have

$$\alpha = \quad K=\frac{v_s-v_c}{v_c}\times\frac{1}{t_s-t_c}. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From (1) and (2) we get

$$K=\frac{l}{L_c}\times\frac{1}{t_s-t_c}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

EXERCISE 11.

TEMPERATURE COEFFICIENT OF AIR PRESSURE.

The air to be tested is contained in a bulb *A*, Fig. 24. To make sure that this air is dry, *A* is exhausted while immersed in boiling water and is later allowed to fill up with air which enters through a drying bottle; and this process is repeated a number of times.

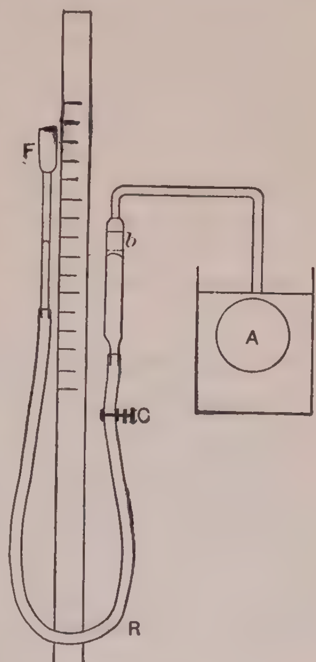


Fig. 24.

After the drying process is completed, a rubber tube *R*, containing mercury, is attached to the glass stem which leads to *A*. On the enlarged part of this stem is fixed a straight-edged band, *b*, of white paper, the lower edge of which marks the height at which the inner end of the mercury column is to be kept whenever the pressure of the air in *A* is measured. This insures that the volume of the confined air shall be always the same, whenever its press-

ure is measured, provided we may neglect, as we shall, the small expansion of the glass which holds the air.

Immerse *A* in water at a temperature t_c, not very different from the temperature of the room, and then, by raising or lowering the funnel *F*, bring the tip of the inner end of the mercury column to the bottom of *b*. Then measure the difference in height of the two ends of the mercury, and call the excess of height of the outer end h_c.

Read now the height of the barometer column, and call this *B*.

The pressure to which the air in *A* is now subjected is $B+h_c$. Call this p_c.

Remove the cold water in *V*, substitute for it boiling water just sufficient to cover *A*, and keep this water boiling. Raise *F* until the inner end of the mercury column is once more at the level of the bottom of *b*. Then measure the excess of height of the outer end, and call this h_s.

The pressure which the air in *A* now bears is $B+h_s$. Call this p_s.

Let t_s stand for the temperature of the boiling water, to be found from the barometric pressure by the use of a steam table.

Let p_0 be the pressure, reckoned in cm. of mercury, which the air in *A* would exert at its present volume if its temperature were 0° C.

Let α be the temperature coefficient of increase of pressure of air at constant volume.

Then, by the ordinary definition of α, we have

$$p_c=p_0(1+\alpha t_c), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$p_s=p_0(1+\alpha t_s), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

from which equations α can be found by eliminating p_o.

Note: Care must be taken to have in the mercury no air bubbles sufficiently large to break the column. If the confined air A is allowed to cool before F is lowered, some of the mercury is likely to run over into A. It is well to have on R a pinch-cock, C, the closing of which will greatly facilitate certain operations, for example the attachment or detachment of R with respect to A.

In the estimation of t_s some account should be taken of the fact that the water touching A is under a pressure somewhat greater than the atmospheric pressure, and therefore in boiling has a somewhat higher temperature than the water just at the surface. If d is the depth of the centre of A below the surface of the water t_s may be taken as the boiling temperature corresponding to a pressure $B + \frac{d}{13.6}$.

EXERCISE 12.

THE PRESSURE-TEMPERATURE CURVE OF WATER VAPOR.

The apparatus, as indicated by Fig. 25, is essentially the same as that invented and used by Regnault for the same investigation which we here undertake.

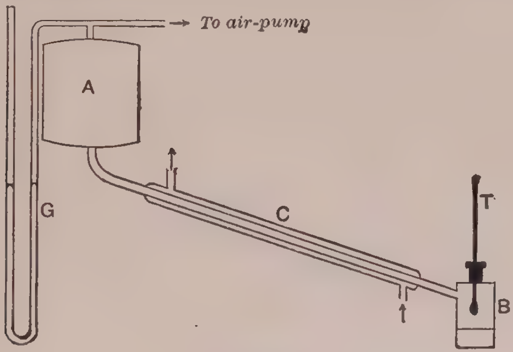


FIG. 25.

B, a thick-walled boiler, is partly filled with water, which is made to boil under various conditions of pressure. *T*, a thermometer which extends into *B* through an air-tight stuffing-box, gives the temperature of the steam formed over the water. *A* is a large air chamber, the object of which is to assist in keeping the pressure within *B* constant during the taking of an observation. Steam coming from *B* toward *A* is condensed in the connecting

tube, which is surrounded by a jacket transmitting cold water, and the resulting water trickles back into B; so that the boiling can be kept up for hours, if need be, without increasing the amount of steam or decreasing the amount of water in the apparatus.

Force air into A till the mercury pressure-gauge, G, shows that the pressure within is about 50 cm. of mercury greater than the atmospheric pressure, then make the water boil in B. Read the excess of pressure and call it p. Read the barometric pressure and call it P. Read the temperature of the steam, t. Record the total internal pressure and the observed temperature under the headings $P+p$ and t, respectively, as the beginning of a table of observations.

Let the pressure fall about 10 cm.; then, after equilibrium is established, read and record pressure and steam temperature as before.

Continue in this way, diminishing p about 10 cm. at a time, till it finally has so large a negative value that the internal pressure is not more than 30 or 40 cm., recording the readings at every stage.

Take a sheet of mm. coordinate paper about 30 cm. square and, beginning near the left-hand lower corner, draw a straight line to the right. Mark the point of beginning 0°, the point 1 cm. to the right 5°, the point 2 cm. from the start 10°, and so on; and write the word *Temperature* alongside this line. From the same starting point draw another line towards the top of the paper, at right angles with the first line. Mark the initial point 0 cm., the point 1 cm. up 5 cm., the point 2 cm. up 10 cm., and so on. Write the word *Pressure* alongside this line.

Take the first numbers in the temperature column of the table of observations, find the point on the temperature line which corresponds to it, then go straight up the paper to a height which represents the first $P+p$ in the table, and there put a dot. The position of this dot represents the temperature and the pressure of the steam in the first condition observed.

Place on the paper in the same general way a dot for each of the other conditions observed.

By the use of a thin straight-edged strip of metal or of hard rubber draw a smooth curve passing, as nearly as may be, through all of these dots. This will be, according to the investigation here made, the pressure-temperature curve of water vapor; and from it the pressure corresponding to any temperature or the temperature corresponding to any pressure, within the range of the observations, can be found.

It is from such studies as this that some of the data for "steam tables" are obtained.

EXERCISE 13.

MIXTURES OR SOLUTIONS.

Mixture of Air and Saturated Vapor.

It is a familiar fact that a volatile liquid will evaporate more rapidly into a partial vacuum than into a space containing air at full pressure. The following experiment is intended to show whether the final vapor pressure of the liquid, when plenty of time is given for the evaporation, is much affected by the presence of air pressure above the liquid.

Take two glass tubes of uniform bore, each about 1 m. long and about 1 cm. in internal diameter, and each closed at one end. Hold each tube with the open end upward and pour in mercury until one, which will be called No. 1, is filled to a level about 0.5 cm. below the top and the other to a level just 10 cm. below the top, taking care to have the mercury column in each tolerably free from air bubbles.

Into the fuller tube, No. 1, pour ether till it is completely filled; then press a finger over the top, invert the tube, put the lower end under mercury, then remove the finger. Immediate evaporation of much of the ether occurs and the mercury column falls to a height much less

than that of the barometric column. Leave the tube in this condition for further observation.

Take the barometric pressure, B.

Take the temperature, t, of the air in the other tube, then close this tube with a finger, invert it, place its lower end under mercury, remove the finger, then, with a pen-filler having a curved beak, force a few drops of ether into the tube, taking care to let no air enter with it. The ether rises to the top of the mercury, evaporating more or less on the way, and the pressure of the vapor, added to that of the imprisoned air, forces the mercury column down some distance. Considerable time will be required, however, for all the air space above the mercury to become fully permeated by the vapor, and therefore the tube should be left to itself for perhaps an hour before final observations are made on it, the temperature of the room meanwhile remaining nearly constant.

After a fixed condition has been reached in each tube, read:

B', present height of barometric column;
h_1, " " " mercury column in No. 1;
h_2, " " " " " " " 2;
L, " " " air column in No 2;
t', " temperature of air near tubes.

Sum of air and vapor pressures in No. 2 $= B' - h_2$.

Air pressure in No. 2 $= B \times \dfrac{10}{L} \times \dfrac{273+t'}{273+t}$.

Vapor pressure in No. 2 $= (B' - h_2) - B\dfrac{10(273+t')}{L(273+t)}$.

Vapor pressure in No. 1 $= B' - h_1$.

Find, in numerical terms, the vapor pressure in each tube and compare the two values.

Freezing-point of Solutions.

The liquid in which the solution occurs is called the solvent. The solvent here used is water.

The substance dissolved is called the solute. Four solutes will be used, of which two, sugar and alcohol, are non-electrolytes, non-conductors of electricity, in aqueous solutions, while the other two, sulphuric acid and common salt, are electrolytes when in water.

In general a solution freezes at a lower temperature than the pure solvent; the difference, ΔT, is called the lowering of the freezing-point. According to the theory of solutions the following formula should hold for non-electrolytic aqueous solutions:

$$\Delta T = \frac{T_0^2}{L} \times \frac{w}{W} \times \frac{1.98}{\mu}.$$

In this formula

T_0 = freezing temp., centigrade absolute, of water = 273;
L = latent heat of melting of ice = 80 approx.;
w = number of grams of solute in solution;
W = " " " " solvent in " ;
μ = molecular weight of solute.

Solutions of a strength indicated below may well be used:

Solute.	μ	$w \div W$.
Cane sugar........	342	0.0855 $(=\frac{1}{4} \times 342 \div 1000)$
Alcohol............	46	0.0230 $(=\frac{1}{2} \times 46 \div$ ")
Sulphuric acid......	98	0.0490 $(=\frac{1}{2} \times 98 \div$ ")
Common salt.......	58.5	0.0293 $(=\frac{1}{2} \times 58.5 \div$ ")

A rather sensitive thermometer should be used, as the change of freezing-point to be observed will perhaps not be more than one or two degrees.

Put the thermometer into a test-tube with sufficient clear water to cover its bulb, hold the test-tube in a freezing-mix-

ture, and, stirring the water continually with the thermometer, note the temperature at which ice begins to form.

Put the same thermometer, after wiping it, into another test-tube containing enough of one of the solutions to cover the bulb, then find the freezing-point of this solution, just as that of the clear water was found.

Compare the ΔT thus found with that indicated by the formula given above.

Test each solution in turn.

See whether the formula holds as well for the electrolytic solutions as for the others.

EXERCISE 14.

THERMAL CONDUCTIVITY OF IRON.

The coefficient of thermal conductivity of a solid, if we may regard its conducting power as independent of the mean temperature, is defined by the formula,

$$k=\frac{Q\times d}{(t-t')AT},$$

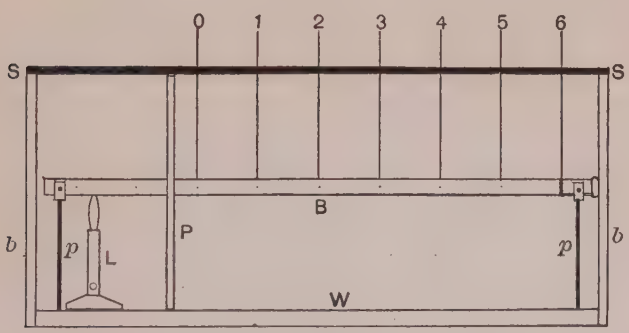


FIG. 26.

where A is the area of either face of a relatively broad, thin sheet of the solid,

d is the thickness of the sheet,

t is the temperature of the warmer face,

t' is the temperature of the cooler face,

Q is the amount of heat transmitted in steady flow through the sheet in T seconds.

Such a sheet as is described above, if actually existing free, is not well adapted for experiments on conducting power, because of the very great difficulty of determining t and t'. In the method devised by Forbes of Edinburgh the sheet of metal studied is part of a long bar, so much of the bar as is included between two imaginary transverse planes lying very near each other. This method will be here used, because of its comparative simplicity, though it is not a very good method for accurate work.

The iron or soft-steel bar B, Fig. 26, is 72 cm. long and 2 cm. square. It is carried on two brass pillars, p and p, rising from a base-board W, the contact between B and its supports being such as is indicated in Fig. 27.

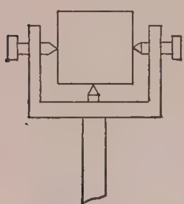


Fig. 27.

B is divided into 9 sections, each 8 cm. long, and at the centre of each section a hole about 0.8 cm. wide is drilled nearly through the bar, large enough to inclose the slender bulb of a thermometer and a packing of mercury, to make good contact between the bulb and the bar. A brass strip SS, carried by brass uprights b, and having holes corresponding to those in B, serves to keep the thermometers 0–6 upright and secure. P is an asbestos partition to shield the observed part of the bar from the direct action of the flame of the lamp L and from the hot gases. A small wad of cotton-wool is placed between the cool end of B and the adjacent b.

We shall take as the object of our especial attention a cross-slice of the bar B 0.01 cm. thick, situated exactly half-way between thermometer 0 and thermometer 1 in Fig. 26. The thickness of this slice is the d of our for-

mula for k. The A of that formula is the area of cross-section of B. The T we shall take as one second. It remains for us to determine $(t-t')$ and Q. We shall consider $(t-t')$ first.

In a large room apply, as in Fig. 26, a small constant flame,* taking precautions against drafts, until the temperature of each thermometer becomes practically stationary, the reading of No. 0 being about 110° C. At least two hours will probably be required to reach this condition. Then note and record the reading of each thermometer on the bar; and record also the temperature of the room, as indicated by a thermometer hanging in the air several feet distant from B.

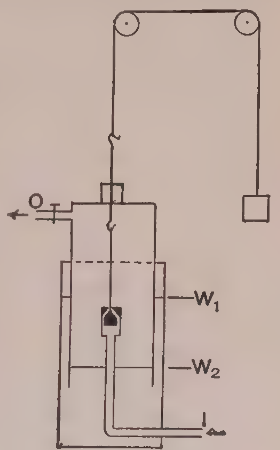


FIG. 23.

On a sheet of mm. coordinate paper about 40 cm. long and 30 cm. wide draw near the bottom a line 36 cm. long, to represent, on a scale of 1 for 2, the length of the bar. On this line, see Fig. 29, mark points, 4 cm. apart, corresponding to the positions of the thermometer-bulbs 0–6. From each of these points draw a perpendicular, the length of which shall represent, on a scale of 1 cm. for 5 degrees, the excess of the temperature of the point above the temperature of the room. Through the tops

* If a gas-flame is used, the gas should come to the lamp through a small pressure-regulator. See Fig. 28. I is the inlet for the gas; O is the outlet; W_1 is the water-level in the outer vessel; W_2 the water-level in the inner vessel.

of all these perpendiculars, as nearly as may be, draw, as in Exercise 12, a smooth curve.

Draw two other perpendiculars, corresponding to the dotted vertical lines in Fig. 29, one from a point 1 cm. to the right from perpendicular 0, the other from a point

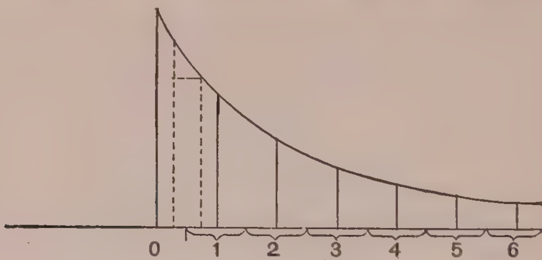


FIG. 29.

2 cm. farther to the right. Measure in cm. the difference of height of these two new perpendiculars, and ~~divide~~ mul this difference by 5. The quotient, which we will call Δ, is the difference in temperature of two points 4 cm. apart on B, these two points being so placed as to have the particular slice of the bar which we are especially interested in midway between them. We can without much error assume that the gradient of temperature midway between these two points is the same as the mean gradient from one point to the other. Accordingly, we can find the $(t-t')$ which we need by dividing Δ by 400.

We have next to consider Q. This is the amount of heat transmitted by the slice in question per second to the parts of the bar lying farther down; and, as these parts of the bar are not, if the state of steady flow aimed at has been attained, changing in temperature, Q must be equal to the heat given out per second by these same parts, the sections 1–6 indicated in Fig 29. This heat

is given out mainly by radiation and by air convection, and if we can find the rate at which each of the isx sections mentioned gives off heat in these ways, we can find Q. The necessary data for this determination could be obtained by heating the whole length of B to 100° C., and then, after removing the source of heat, and putting non-conducting wads of cotton between the ends of the bar and the uprights b in Fig. 26, noting the rate of fall of temperature of the bar at various stages in its cooling.

This operation, however, is more laborious than interesting, and accordingly the following table is given, which has been derived from observations on a bar similar in material and size to that used in this exercise. Each number in the second column gives the rate, as a fraction of a degree per second, at which such a bar, left to itself after being uniformly heated, would be cooling at any instant by radiation and air convection from its sides, if the excess of its temperature above the temperature of the room at the instant were that indicated by the corresponding number in the first column.

Excess Temperature.	Fall per Second.
10°	0°.0050
20°	0°.0101
30°	0°.0154
40°	0°.0207
50°	0°.0262
60°	0°.0318
70°	0°.0377
80°	0°.0440

The rates of fall for intermediate temperatures can be found by simple interpolation.

Assume that in our case of steady flow, Figs. 26 and

29, the mean temperature of the 8 cm. section numbered 1 was that indicated by thermometer 1. By means of the table given above, taking into account the volume, density (7.8?) and specific heat (0.114?) of the section, find the amount of heat given out per second from the sides of this section. Call this Q_1. Make a similar calculation for each of the other five sections concerned. The Q which we need for the determination of k is

$$Q=Q_1+Q_2+Q_3+Q_4+Q_5+Q_6.$$

EXERCISE 15.

HEATING AND VENTILATION.

Examine in operation the system of heating and ventilation in some large, well-equipped building, and make notes and sketches of the most important features.

At the Jefferson Physical Laboratory at present we make free sketches of the following subjects:

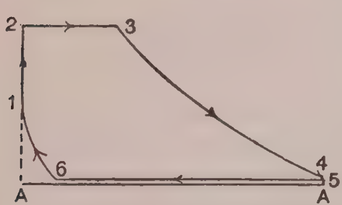
Seguin boiler, longitudinal section,
" " , end view.
Horizontal steam radiator,
Vertical " "
Contrivance for heating a draft of air.

EXERCISE 16.

HORSE-POWER OF A STEAM ENGINE.

The engine used in this Exercise at the Jefferson Physical Laboratory is non-condensing, has a horizontal cylinder 10 inches in diameter, a stroke of 15 inches, an ordinary fly-ball governor, and a Myers slide-valve. The valve-chest is on the side of the cylinder, so that a diagram of a horizontal section shows the piston, ports, main valve and cut-off valve, with the device for adjusting the time of cut-off. This diagram is studied by the class before the engine is taken in hand, and is copied with more or less precision by each student.

Fig. 30 shows a typical "indicator-diagram" for such an engine as that just described; that is, an automatic-



1 to 3 = admission;
3 = cut-off;
3 to 4 = expansion;
4 = release;
5 to 6 = exhaust;
6 to 1 = compression;
A to A = line of atmospheric pressure.

FIG. 30.

ally-made record of the pressure in one end of the working cylinders during one complete forward and back stroke of the piston, or one revolution of the fly-wheel.

In this diagram vertical distances represent pressure and horizontal distances represent parts of the stroke.

Such a diagram the class is presently to take from the engine in action. Its use will be seen in the calculation of horse-power, the formula for which will now be derived.

Let A = area, in sq. in., of piston-face.*

Let P = mean press., lbs. per sq. in., on piston-face during forward stroke, (2) to (5) in Fig. 30.

Let p = mean press., lbs. per sq. in., on piston-face during back stroke, (5) to (1) in Fig. 30.

Let L = length of stroke in feet.

Work done by steam in one end of cylinder during one forward stroke $= P \times A \times L$ ft.-lbs.

Work done against steam in same end of cylinder during one back stroke $= p \times A \times L$ ft.-lbs.

Net work done by steam in one end in one revolution of main shaft $= (P - p)AL$ ft.-lbs.

Net work done by steam in this end in one minute, with N revolutions, $= N(P - p)AL$ ft.-lbs.

For both ends of the cylinder, if we take for $(P - p)$ the mean value for the two ends and for A the mean value for the two ends (see foot-note), we have, remembering that one horse-power is 33,000 ft.-lbs. per minute,

$$\text{``Indicated horse-power''} = \frac{(P-p)A \times 2NL}{33,000}.$$

This is called the "indicated" horse-power because the factor $(P - p)$ is obtained from a study of the indicator-diagram. $(P - p)$ is the mean difference of pressure indicated by the forward-stroke line and the backward-stroke

* For the "head end" of the cylinder A is the area of cross-section of the cylinder; for the "rod end," or "crank end," of the cylinder A is equal to the area of cross-section of the cylinder *minus* the area of cross-section of the piston-rod.

line in a diagram like Fig. 30. To find this difference we have to find in sq. in. the area inclosed by the closed line of the diagram, divide this by the length of the diagram, which is the length of the atmospheric line, and multiply the quotient by a factor depending on the stiffness of the indicator spring.

In the engine-room the external parts of the engine are examined, the working of the oil-cups is explained, the method of starting and stopping is shown, the indicators are exhibited and attached to the cylinder. Preliminary diagrams are taken with various lengths of admission and with variation from light to full load.

With a low boiler-pressure, about 25 lbs. per sq. in. above atmospheric pressure, and a moderate speed, 80 to 100 revolutions a minute, each student works the indicator in turn, takes a "card, or diagram, and counts N, the number of revolutions per minute. He measures the diagram with the use of a planimeter and finds $(P-p)$, after which he calculates the horse-power of the engine for the conditions which he has observed.

EXERCISE 17.

HEAT OF COMBUSTION OF ALCOHOL.

The object of this Exercise is to find, approximately, how many calories are given out in the combustion of 1 gram of ordinary alcohol in air. The alcohol is burned from a lamp *L*, Fig. 31, in a brass combustion-chamber, *C*, submerged in water, to the level *w*, in a metal vessel *V*.

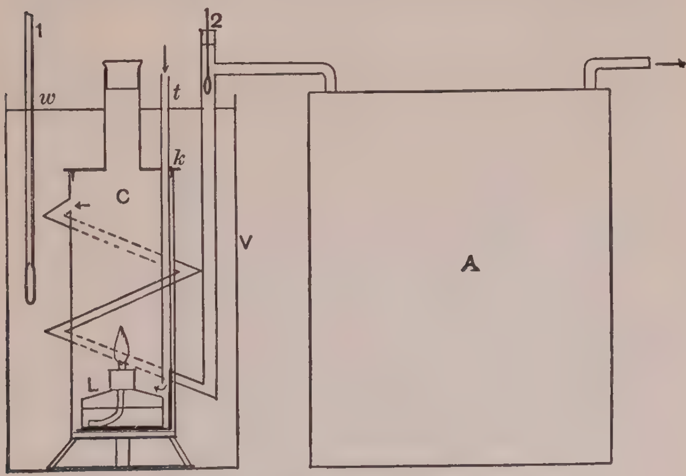


FIG. 31.

Air to support the combustion is drawn by pump action down through a tube, *t*, which pierces and is soldered to

the cover, k, of C. Air and the products of combustion pass out near the top of C to a brass tube, which makes a spiral of five or six turns about C, and then go, by the way indicated in Fig. 31, past the bulb of a thermometer, 2, and through an air-chamber, A, intended to steady the flow. Another thermometer, 1, has its bulb immersed at mid-height in the water contained in V. A ring-shaped stirrer, which surrounds C and is moved up and down in V during the combustion of the alcohol, is not shown in Fig. 31.

Find, within a few grams, the weights of the vessels V and C, both of which will have practically the same rise of temperature as the water, and from this weight and the specific heat of the metal composing them calculate the thermal capacity, or "water-equivalent," W_m, of the metal to be heated.

Find by a preliminary trial the amount of water, about 5000 grams, needed to cover C in V, as in Fig. 31, cool this amount of water to a temperature seven or eight degrees below the temperature of the room, pour it into V, weigh V and the water, and so find the weight of the water. Record this weight as W. Bring the water in V to a temperature about five degrees below that of the room.

Half-fill the lamp with alcohol and find, to the nearest tenth of a gram, the weight of lamp and contents. Call this w.

Make the top of the wick project about 0.7 cm. and spread it somewhat. Place the lamp on the platform at the bottom of the tube leading down from the cover of C, lower it, unlighted, into C, push down the cover of C and seal it in place water-tight by rubbing tallow into the crevice.

Stir the water thoroughly in V and take its temperature, t_1, to the nearest tenth of a degree. Set the pump in operation, and immediately light the lamp by means of a burn-

ing wisp of cotton saturated with alcohol, let down by a wire through the central hole in the cover of C. Then close this hole. Note the time of lighting the lamp.

Take every two minutes the temperature of the water, after thoroughly stirring it, the temperature of the issuing air, by thermometer 2, and the temperature of the air of the room at large, by a third thermometer, and record these readings in a table like that indicated below:

Time.	Th. 1.	Th. 2.	Th. 3.
0	......	......	......
2 min.....	......	......	......
4 "	......	......	......
etc........	......	......	......

The operation should be continued until the water has risen from about 5° below to about 5° above the temperature of the room. If the wick of the lamp and the draft are right, this will require about 20 minutes. Of the temperature observations, only the first and the last with Th. 1 are of much importance in the final result; but the other observations are of some value in the discussion of certain possible corrections. Moreover, observation at regular intervals shows whether the combustion is going on as it should.

When the combustion has continued long enough, stop the pump and allow the flame to go out, then stir the water thoroughly and take a final reading, t_1', of its temperature. Immediately remove the lamp from C, weigh the lamp and its contents again as carefully as before, and record this weighing as w'.

The main result is now found very simply as follows:

$$\text{Heat yielded by 1 gram of alcohol} = \frac{(W+W_m)(t_1'-t)}{w-w'}.$$

There are, however, a number of rather troublesome correc-

tions, which it is not unprofitable to glance at. For example:

1st. Heat absorbed by rise of temperature of lamp and contents. This quantity is not likely to be very large, perhaps a few hundred calories.

2d. Heat carried away from the apparatus by the stream of air and gases in excess of the heat brought in by the air. Thermometer 2 usually indicates that the escaping stream has a temperature very little, if any, higher, perhaps somewhat lower, than the temperature of the water in V, which in the mean is very nearly the same as the temperature of the air of the room at large. One might, therefore, conclude that the air passing through the apparatus carried away, on the whole, no more heat than it brought in. The fact is, however, that the air is cooled, perhaps several degrees, by its expansion in entering the apparatus, so that it does take up from C and carry away an appreciable amount of heat. Fifteen liters of air drawn through per minute give a strong draft. In twenty minutes, if we assume that the air leaves the apparatus 5° warmer, in the mean, than it enters, this draft would carry off about 400 calories, probably less than 1 per cent. of the total heat given out in the combustion.

This Exercise does not undertake to find the heat of combustion of pure alcohol, but only of the commercial article, which contains some 10 per cent. of water. The density, at some known temperature, of the alcohol used should be taken, from which, by the use of the proper table of values, its strength in per cents., by weight, of pure alcohol can be found.

EXERCISE 18.

SOLAR RADIATION, ETC.

Heat Received from the Sun.

The object of this experiment is to find out, approximately, how much energy of solar radiation is received per second by a surface of given area exposed squarely to such radiation. The instrument used for this purpose is called a *pyrheliometer*. Fig. 32 shows it in simple form.

B is a blackened disk, about 10 cm. in diameter, which forms the front of a thin-walled brass box about 1.5 cm. in depth. From the centre of the back of this box a short hard-rubber tube *r* leads to a brass tube *b*, which can be fixed at any desired angle of inclination in a clamp *C* carried by a pillar *P*. At the lower end of *b* is a disk, *D*, of the same size as *B*. *T* is a sensitive thermometer of low range, the bulb of which is at the centre of the box; it extends into the box through a stopper which can be removed. The surface of the box is nickel-plated or silvered and, except on the face *B*, is polished.

Take out *T* and the stopper, and remove the box from *r*. Weigh the box, nearly fill it with water, at a temperature near that of the air in shadow out of doors, and weigh it again thus filled; replace *T* and the stopper and

reset the box on *r*. Place the apparatus in the sunshine out of doors, and adjust it so that the shadow of *B* will just cover *D*. Place about half a meter from *B* a screen just large enough and so adjusted as to put *B* completely in shadow. With this condition of things note the tem-

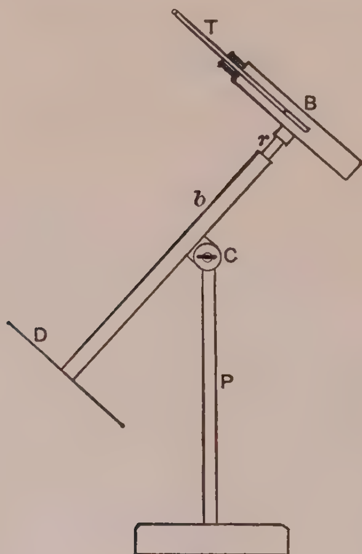


FIG. 32.

perature by *T*, wait 5 minutes, and note the temperature again. Then remove the screen which shades *B*, readjust the apparatus so that *D* will be just covered by the shadow of *B*, and with this condition of things record the temperature every 5 minutes for 20 minutes. Then replace the screen which shaded *B*, wait 5 minutes more, and again record the temperature.

The observations should be recorded in a table like the following:

Condition.	Time.	Temperature.	
B in shade	0.0	.....	
" " " / *B* in shine	5 min.	.....	
" " "	10 "	.....	
" " "	15 "	.....	Rise in 20 minutes. .....
" " "	20 "	.....	
" " " / *B* in shade	25 "	.....	
" " "	30 "	.....	

To find the effect of the direct sunshine, estimate from the first and lust observations, made with B in shade. what the rise of temperature would have been in 20 minutes with B in shade, and subtract this amount from the actual rise during the 20 minutes of the illumination of B. Call this corrected rise t.

If R is the radius of B, if W is the weight of the water, and W_m the water-equivalent of the box itself, we have, as the amount of heat absorbed per second from direct solar radiation by 1 sq. cm. of blackened surface,

$$k=\frac{(W+W_m)\times t}{\pi R^2\times 20\times 60}\text{ cal.}$$

Remembering that one horse-power is equivalent to about 178 calories per second, we can readily calculate from our observations how large an area exposed to direct sunshine, under the conditions prevailing during this experiment, would absorb energy at the rate of one horse-power. Of course, a much larger area than the one thus found would be needed to give the heat required for running a one-horse-power engine.

Heat from Mechanical Energy.

Use the Tyndall apparatus for making a small amount of water boil by heat generated by friction.

EXERCISE 19.

STATIC ASPECTS OF ELECTRICITY.—I.

With catskin and gutta-percha or hard rubber, silk and glass, and pith-balls suspended by silk threads, illustrate the following facts:

That there are two states of electrification, which may be called + and − respectively.

That like electrifications repel, and unlike electrifications attract, each other.

That each of two bodies rubbed together acquires electrification opposite to that required by the other.

Illustrate electric conduction by taking the charge from a suspended pith-ball by touching it with a wire or with a moistened thread.

With a gold-leaf electroscope, and gutta-percha charged ("negatively") by rubbing with catskin, illustrate the state of temporary charge produced by "static electric induction."

Give a permanent charge to the gold leaves by touching the knob or plate of the electroscope with a finger while the charged rod is held near and removing first the finger and then the rod. Discuss the question whether the charge thus left on the leaves is like or unlike the charge on the gutta-percha rod.

Illustrate static electric induction further by use of the electrophorus, and show that an indefinitely large number of charges can be given to the metal plate by the operation of this apparatus without recharging the non-conducting plate.

Show by use of "proof-plane" and electroscope that no charge can be taken from the inner surface of a hollow charged metal sphere.

Illustrate with a simple form of Faraday's "ice-pail experiment" the incompressible-fluid behavior of electric charge.

Illustrate the action of an electric condenser by counting the number of audible sparks which can be given by repeated use of an electrophorus to the lining of a dissecting Leyden jar, 1st, when this lining is removed from the jar and is placed on an insulating support; 2d, when the lining is in place in the jar and the other coating of the jar is connected with the ground. Compare also the loudness of the discharging sparks in the two cases.

Show the ordinary experiment with the dissecting Leyden jar. That is, charge the jar, remove the coatings, taking great care not to touch both of them at the same time until they are free from the jar, touch the coatings together, replace them, cautiously, on the jar, then connect them with a discharging-rod.

With apparatus like that indicated by Fig. 33 illustrate the difference of specific inductive capacity of glass and air. In this figure *a* and *b* are disks of brass, about 15 cm. in diameter, separated by small pillars of hard rubber about 1 cm. tall; *c* and *d* are disks similar to *a* and *b*, but separated by a plate of varnished glass, *g*, of

a thickness equal to the height of the blocks which separate a and b. All the uprights marked r leading up from the base-board are of hard rubber. The rods m_1 and m_2 are of metal, one connected with the + pole, the other with the − pole, of a Toepler-Holtz machine or some equivalent apparatus. Thin metal springs run from m_1 to a and c and from m_2 to b and d, but the ap-

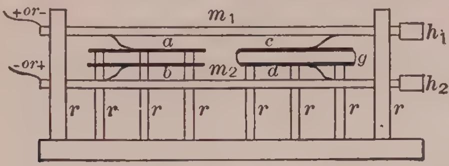


FIG. 33.

paratus is so contrived that a slight turn of the hard-rubber handles h_1 and h_2 will withdraw these springs from the plates. The hard-rubber parts should be freshly cleaned to prevent leakage over their surfaces.

Charge all the plates simultaneously, stopping before there is danger of sparking across from a to b; then break the contacts and, as promptly as may be, discharge each pair, a and b, c and d, by means of a discharging rod, and compare the two sparks in regard to loudness and brightness.

Try the ordinary experiment with a "unit jar."

First Law of Electrostatics. Du Fay's Law
" " " " . Coulomb's Law

$F = \frac{1}{c}\frac{q q'}{r^2}$ [illegible]

Quick

Brush discha

Study of static machine

Screening effect of

Volta's cells

EXERCISE 20.

STATIC ASPECTS OF ELECTRICITY.—II.

1" 16 thicker

Show the maximum length and penetrating power of the spark from the most powerful static induction or frictional machine available.

Show, with the ordinary apparatus for this purpose, the piercing of a thin sheet of glass by the discharge from a Leyden jar charged by the machine.

Show that the noisy discharge of a Leyden jar by a spark through a pinch of gunpowder lying between two metal balls on a glass plate will not ignite the powder, but that, if some inches of wet string are put in the path of the discharge, between one coating of the jar and one of the balls on the glass, the discharge will be noiseless, which means that it will be comparatively slow, and will ignite the powder.

Show the "brush discharge" from the machine in a darkened room, noting the tendency to discharge from points.

With a proof-plane, furnished with a short flexible wire projection, and a gold-leaf electroscope test the nature of the charge at various significant parts of the machine in action. (See § 366 of Hall and Bergen's Text-book of Physics, 3d edition.)

Run a wire from the outer coating of one of the Leyden jars of this machine to a point about 0.1 cm. distant from the outer coating of another jar of the machine, the same two coatings being actually connected by metal in the construction of the machine, and show that, when a strong discharge takes place between the poles, a small spark leaps across the air-space left between the wire and the jar. This illustrates what is called alternative-path discharge between two bodies. Of course, the greater part of the momentary flow from one outer coating to the other passes by way of the regular metallic path which connects them.

Arrange two sheets of tin, *A* and *B*, each about 50 cm. square, as they are shown in Fig. 34, insulated from each

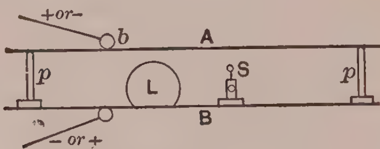


Fig. 34.

other by pillars of glass or hard rubber, such as *p*, *p*. Place on *B* a large ball, *L*, reaching within about 1.5 cm. of *A*, and a small ball, *S*, adjustable at various heights on a metallic support, reaching not quite so near to *A*. Connect *A* with one pole of the electrical machine and *B* with the other pole, and work the machine, the poles being separated several centimeters. Note that the discharges pass between *S* and *A* rather than between *L* and *A*. Then, while the machine is operated, slowly separate the ball *b* from *A*, keeping it all the time connected with the pole. Note whether at any stage of this separation the discharge passes between *L* and *A* rather than between *S* and *A*. This experiment has some bearing on lightning

strokes, *B* representing the earth and *A* representing a thunder-cloud.

Show that a gold-leaf electroscope placed in a wire cage is undisturbed by strong electric sparks passing between the cage and a body outside it.

Show that a considerable charge can be given to a gold-leaf electroscope from a battery of 50 zinc-water-copper cells joined in series, provided Volta's condensing deivce is used.

Try a Lippmann electrometer of very simple form with the same battery of cells.

EXERCISE 21.

STATIC ASPECTS OF ELECTRICITY.—III.

Atmospheric Charge.

Study by means of a quadrant electrometer and a water-dropping contrivance the state of charge of the out-door air.

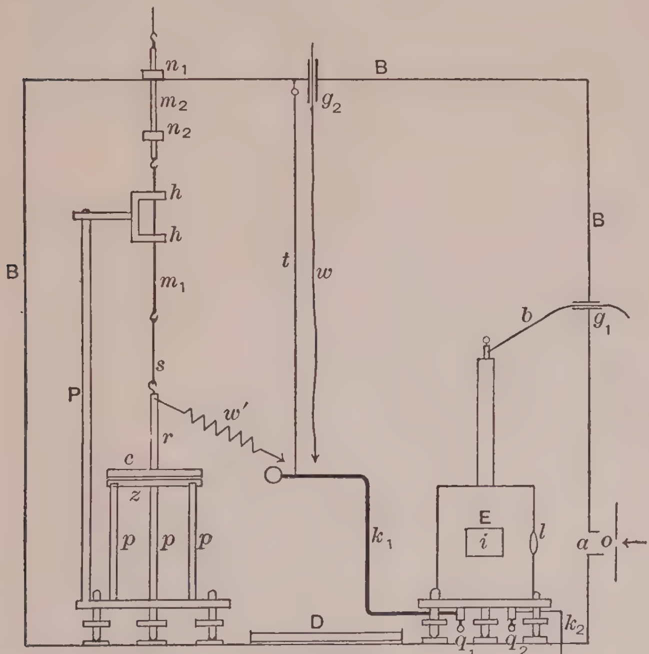


Fig. 35.

The water reservoir can be placed on a shelf over a window, the long metal tube from it reaching through an in-

sulating glass tube inserted in a hole bored through the window-casing.

The electrometer, E, Fig. 35, should be placed in a closet, B, lined everywhere with tinfoil connected metallically with the water-pipes or gas-pipes of the laboratory, and containing a broad shallow dish, D, holding strong sulphuric acid to keep the air dry. A Welsbach burner or powerful incandescent electric lamp sends a beam of light through the openings o and a and the lens l to the mirror attached to the needle, whence it is reflected to an inclined plane mirror i, on the front of E, which sends it up through a long slit in the top of B to a scale fastened above. The needle is kept charged through a wire b leading, through the glass tube g_1, from one pole of a zinc-copper-water battery of fifty cells in series, the other pole of the battery being grounded. The case of E is grounded by way of its feet, which rest on the tin lining of B. One pair of quadrants is grounded by way of the binding-post q_2 and the wire k_2. The other pair of quadrants is connected, by way of the binding-post q_1, with the stout copper wire k_1, the outer end of which is supported by a silk thread, t. The copper wire w, leading from the water-dropper by way of the glass tube g_2, is connected, directly or indirectly, with k_1, when the condition of the external air is to be tested. The indirect way of making this connection is indicated by Fig. 36, where b is a wooden block fastened to the back of B; g is a glass tube firmly fixed in b; c_1 is a copper rod fastened with sealing-wax into the end of g; r is a thin ribbon of copper, about 2.5 cm. wide and 4 cm. long, soldered to c_1 and to another copper rod, c_2; s is a silk thread; z is a spiral spring; t is a wire lead-

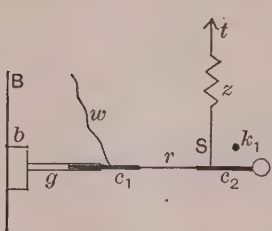


FIG. 36.

ing through a hole in the top of B and fastened at its top to a string by which it may be lifted. The wire w, connected with the water-dropper, is by the lifting of t brought into metallic contact with k_1, which leads to q_1, etc. A device similar to that shown in Fig. 36 serves to connect k_1 with the earth when it is desired to discharge the quadrants 1.

The sign and, approximately, the magnitude of the charge received from the air can be determined by comparing the deflection which it produces on the scale with that produced when one pole of a galvanic cell of known electromotive force is connected with k_1 by way of the wire w, the other pole being connected with the ground.

The Volta Effect.

Study the "volta effect" between zinc and copper. In Fig. 35 z is a zinc disk, about 10 cm. in diameter and 0.5 cm. thick, with both sides turned plane, resting on metal posts, p, which are in metallic contact with the earth; and c is a similar disk of copper, from the centre of which leads a copper rod, r, connected by way of the thin spiral of copper wire w', and by a device like that shown in Fig. 36, with k_1. The copper disk is suspended by a silk thread, s, attached to a metal rod, m_1, which runs through a guiding hole in the metal arms h, h (supported by the metal post P), and is hooked above to another metal rod, m_2, which runs through a hole in the top of B, just outside of which is a nut n_1, which limits the descent of b and so fixes the distance between c and z.

Make this distance about 0.05 cm., connect c with k_1, ground k_1 for a moment, thus getting rid of previous charge and connecting c conductively with z, break the ground connection of k_1, raise c, by means of a string fastened to

the top of m_2, as far as the nut n_2 will permit, and note the direction and magnitude of the permanent deflection on the electrometer scale.

Then lower c, still connected with k_1, to its first position, connect k_1 with one pole of a galvanic cell of known e. m. f., the other pole being connected with the ground, break connection of k_1 with this cell as soon as the indicator spot on the scale has come to rest, then raise c as before and again note the direction and magnitude of the permanent deflection of the indicator spot, from the position occupied just before c is raised. If this deflection is greater than the one produced when c was raised before, the battery e. m. f. has been added to the volta effect; if smaller, the battery e. m. f. and the volta effect have been in opposition; in either case, the change of deflection will give an indication of the magnitude and direction of the Volta effect.

Test the Volta effect when the copper is tarnished and the zinc bright, and again when the copper is bright and the zinc tarnished, in order to see that this effect is dependent on the surface condition of the two metals.

$M = a$ [illegible]

$M = k\, i_a\, t$

$i_a = \dfrac{M}{kt}$

$F\,(\text{dynes}) = \dfrac{2\pi i n}{a}$

k = electro-chemical equivalent

t = time

i_a = current in amperes

a = radius of coil

n = no. of turns

i = current in abs. mag. units

F [illegible] $= m H \dfrac{l}{2} \sin\phi$

$F \dfrac{l}{2} \cos\phi = m H \dfrac{l}{2} \sin\phi$

$H \dfrac{\sin\phi}{\cos\phi} = F \quad \therefore H \tan\phi = F = \dfrac{2\pi n i}{a}$

$i = \dfrac{a}{2\pi n} \times H \tan\phi$

$H \tan\phi = \dfrac{2\pi n}{a} \dfrac{i_a}{10} = \dfrac{2\pi n}{a \times 10} \cdot \dfrac{M}{kt}$

EXERCISE 22.

VOLTAMETER MEASUREMENT OF CURRENT AND OF H.

The formula to be used in getting i, the mean strength of the current in absolute c. g. s. electromagnetic units, during the t seconds of its flow, in which it deposits M grams of copper from a copper sulphate solution, is

$$i = M \div (t \times 0.003277). \quad . \quad . \quad . \quad . \quad (1)$$

The quantity 0.003277 is the so-called "electrochemical equivalent" of copper. The formula connecting this i with H, the horizontal intensity of the earth's magnetic force, is

$$H = 2\pi ni \div R \tan \theta \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where R = the radius of the windings of a tangent galvanometer;

n = the number of windings used;

θ = the angle of deflection maintained in this galvanometer by the current.

The current flows through the voltameter and the galvanometer simultaneously.

Two voltameters are used for greater accuracy, although only one is strictly necessary, as the deposit should be the same in both. Two galvanometers are used, because it is desirable to determine the value of H in two places in the laboratory, local conditions, gas pipes, etc., often

making this value change several per cent. in a distance of a few feet.

The general arrangement of the apparatus is shown in Fig. 37. S is a storage cell; V_1 and V_2 are the voltameters; C is a commutator, which can not reverse the current in V_1 and V_2, but can reverse it in the galvanometers G_1 and G_2

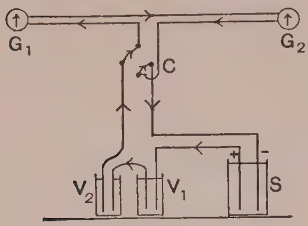


FIG. 37.

The fluid in each V is a solution, of density about 1.15, of sulphate of copper in water, to which about 1 per cent. by weight of strong sulphuric acid has been added. Each voltameter plate, of sheet-copper, is of the general shape shown in Fig. 38, the main part being about 8 cm. long and 6 cm. wide. Fig. 37 shows only two plates in each V, but it is better to have the gain plate in each placed between two loss plates, which may well be somewhat wider than the gain plate. The whole of the main part of each gain plate should be covered by the liquid; and the plates should be held in such a way that the distance between them will not change accidentally, as change of this distance affects the resistance of the circuit and so the strength of current.*

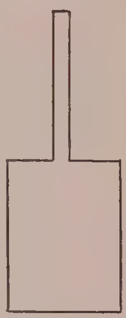


FIG. 38.

All the copper plates should be scoured bright before

* By regulation of the distance between plates or by some other means of controlling the resistance, the current should be made something between 1 and 2 amperes, if the gain plate lies between two loss plates.

use. The gain plates, which alone by their change of weight give a reliable measure of the current, the loss plates acting in a variable way, are then to be treated as follows:

Wash the plate carefully with water under a tap.

Rinse the plate carefully in water containing 5 per cent. (by volume) of strong sulphuric acid. Wet the plate with alcohol, allow it to drip a few seconds, then touch it with a Bunsen flame and burn off the alcohol.

Weigh the plate to the nearest 0.005 gm.

Connect all the apparatus as in Fig. 37, keeping the plates, however, out of the voltameters until the last; then, noting the time, lower the plates into the liquid.

At the end of 2 minutes from the start read each galvanometer and record the readings in a table like that here shown:

Time.	θ Galv. No. 1.		θ Galv. No. 2.	
2 min.	......		......	
4 "	......		......	
6 "	......		......	
8 "	......		......	
10 "	......			
12 "		......		......
14 "		......		......
etc.		......		......
20 "		......		
22 "	......		......	
24 "	......		......	
etc.	......		......	
Means	......	......	......	......
Final means	...... $(=\theta_1)$		...... $(=\theta_2)$	

Go on according to the suggestion given in this table, reversing the commutator quickly at the end of 10 minutes and again at the end of 20 minutes. At the end of 30 minutes promptly stop the current by lifting the plates from the voltameters; take out the gain plates and treat them as follows:

Rinse the plate first in water containing a few drops of strong sulphuric acid to the liter, then rinse it in clear water under a tap.

Wet the plate with alcohol, then burn this off as before.

Weigh the plate as carefully as before.

Take the mean increase of weight of the two plates as the M to be used in formula (1).

Find θ_1, the mean θ for galvanometer 1, and θ_2, the mean θ for galvanometer 2, according to the suggestion given at the bottom of the table.

Find R_1, for galvanometer 1, and R_2, for galvanometer 2, by direct measurement, and count n_1 and n_2, the number of turns used in each respectively.

Find i and find also i_a, the latter being the strength of the current in amperes.

Find H_1, for position of G_1, and H_2, for position of G_2.

$$i = i_1 + i_2$$

$$i r_2 + i_1 r_g + r i + R i = E x$$

EXERCISE 23.

ELECTROMOTIVE FORCE OF A GALVANIC CELL.

In Fig. 39, D is a fresh Daniell cell, the e. m. f. of which is to be found by what is called Poggendorff's method,

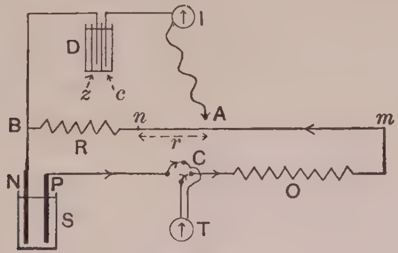


FIG. 39.

z being the zinc plate and c the copper plate. S is a storage cell, of which P is the positive, and N the negative, pole. C is a commutator, which can reverse the current in the tangent galvanometer T but not in other parts of the circuit. T should be in one of the positions, see Exercise 22, for which H is known. O is a resistance, of perhaps 10 ohms, which serves to regulate the current from S but does not enter into the numerical calculations which follow. The part mn is a straight, tautly held, piece of german-silver wire, the resistance of which, some ohms, should be

accurately known. R is another known resistance, some of the coils of a resistance-box, and should be perhaps 8 ohms. I is an astatic galvanometer used not to measure, but merely to indicate the existence or non-existence of a current flowing through it and through D. Such a current will, of course, not exist until the flexible wire leading from I, and shown with an arrow-point, is touched to the wire mn.

With the current from S flowing as it does in Fig. 39, all points on the wire mn have a potential higher than the potential of the point B, and accordingly, when contact is made between I and any point on mn, this difference of potential has a tendency to send a side-current through I and D to B. On the other hand, the natural e. m. f. of D tends to send a current from zinc to copper through the cell. Either of these two opposing tendencies may prevail, according to the circumstances of the case; but, if the current through T is of the right magnitude, it will be possible to find some point of contact, A, on mn such that there will during this contact be no current through I. When this condition is reached, the e. m. f., E, of D is exactly equal to the opposing difference of potential $P_A - P_B$, which we will call Δ.

To find Δ, and so E, after the point A is properly located, we have to find the resistance, r, of the part An of the wire mn, measure i, the strength in amperes of the current through T, and then write

$$E = \Delta = (R+r)i \text{ volts.}$$

The value of i should not be found from one reading only of T. The current through T should be reversed a number of times, so as to give deflections in both directions from the zero position of its needle, and the mean

deflection should be found at last according to the suggestion made at the bottom of the table in Exercise 22.

As slightly different positions of A may be found with these varying conditions of T, the mean of all the several values of An should be taken for the calculation of E.

If time permits, measure the e. m. f. of some other form of cell, the Leclanché, for example.

EXERCISE 24.

RESISTANCE OF AN ELECTROLYTE AND OF A GALVANIC CELL.

SPECIFIC RESISTANCE OF SULPHATE OF COPPER SOLUTION.

J, Fig. 40, is a tall, narrow, cylindrical jar of glass, which is to contain the solution. P_1 is a flat plate of copper which nearly fills the whole cross-section of J, and which is kept

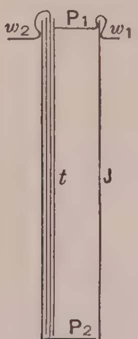


FIG. 40.

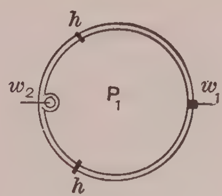


FIG. 41.

in a horizontal position by copper hooks, see Fig. 41, h and h, reaching over the edge of the glass. The wire w_1 is riveted, not soldered, to P_1. P_2 is a plate similar to P_1. The wire w_2, which leads down to P_2 and is riveted to it, is inclosed in a glass or rubber tube, t, from the top of J to the bottom.

Measure carefully with calipers, or by cutting a piece of paper to the right size, the internal diameter of J. Call this D.

Measure the distance from P_1 to P_2 and call this L.

Fill J with a solution of known density of sulphate of copper. Note the temperature of the solution.

By means of a Wheatstone-bridge arrangement measure in ohms the resistance of the column of liquid from P_1 to P_2. Call this resistance R.

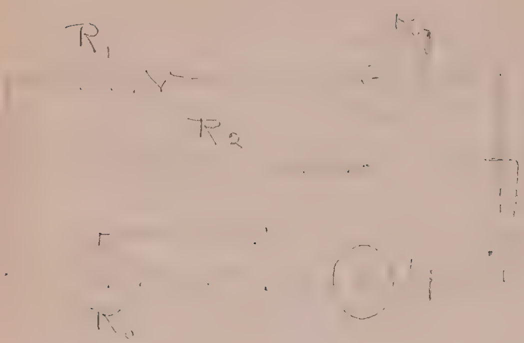
From R, L, and D calculate the specific resistance, r, of the solution.

Compare the value of r thus found with that given by Landoldt and Börnstein, or other good authorities, for a solution of sulphate of copper of the same density and temperature as that here used.

Resistance of a Galvanic Cell.

If one should undertake to find the resistance of a single galvanic cell by means of the Wheatstone bridge, just as the resistance of the column of sulphate of copper was measured, the electromotive force of the cell would make the attempt a failure. It is possible, however, by connecting two like cells, two Daniells, for example, in opposition, so that the e. m. f. of one will be neutralized by that of the other, to measure with some approach to accuracy, by the Wheatstone-bridge method, the resistance of the two thus combined. This resistance is twice the resistance of a single cell.

In making this experiment, as in the use of the Wheatstone bridge generally, care should be taken not to use an unnecessarily large current and not to keep the current flowing unnecessarily long.



$i =$ cu[illegible]

$i_g = i \times \frac{R_2}{[illegible]}$

$i_g = K\,\Pi \quad K = \frac{i_g}{\Pi} = i\,\frac{R_2}{R_1 + R_2 + R_g} \times \frac{1}{\Pi}$

$= iR = i(R_g + R_1) = K\,\Pi\,(R_g + R_1)$

hermoelectric power = P =

EXERCISE 25.

REDUCTION FACTOR OF A SENSITIVE GALVANOMETER; THERMOELECTRIC MEASUREMENTS.

Reduction Factor of Galvanometer.

By this title we shall mean the factor by which one must multiply the deflection observed on the scale of the galvanometer in order to find in amperes the strength of the current which produces this deflection. Call this factor F.

The arrangement to be used for determining F must depend somewhat on the sensitiveness of the galvanometer to which F belongs. It is not likely to be more complicated than the combination indicated by Fig. 42, and perhaps a simpler combination will serve.

In Fig. 42, S is a storage cell; O is a resistance, used to control the magnitude of the main current used, but not

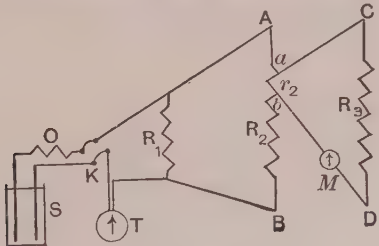


Fig. 42.

entering into the numerical calculations; K is a commutator; T is a tangent galvanometer; M is the galvanometer to be tested. The resistance R_3 is many times as great

as r_2, which is a small part of R_2, and R_2 is many times as great as R_1. The resistance of M, R_m, is supposed known.

The whole current, i, which comes from S, is measured in amperes by means of the tangent galvanometer T. To find exactly what part of this current goes through M is a somewhat laborious though not difficult calculation based on the familiar principles of a "divided circuit." But if, as may well be the case, $R_3+R_m>100r_2$, $R_2>100r_2$, $R_2>100R_1$, we may write, with quite sufficient accuracy for our purpose, as the current through M, expressed in amperes,

$$i_m = i \times \frac{R_1}{R_1+R_2} \times \frac{r_2}{r_2+(R_3+R_m)}.$$

The various resistances should be such that an ample deflection for easy measurement will be obtained with each galvanometer.

If D is the deflection observed on the scale of M, produced by the current i_m, the value of F is found from the relation $i_m = F \times D$.

Study of a Copper-iron Thermoelectric Couple.

In Fig. 43, L is a vessel containing oil at a low temperature, T_l; H is a vessel containing oil at a higher tempera-

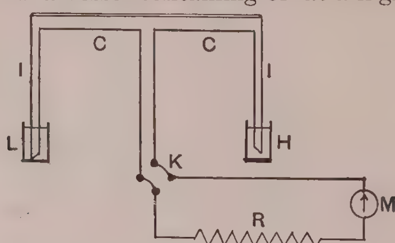


Fig. 43.

ture, T_h; II is a soft iron wire; C and C are copper wires soldered to II; K is a commutator; M is the galvanometer just studied; R is a resistance which can be varied.

Make T_l 20° C., or any other convenient temperature not much above that of the room, and keep this temperature constant, stirring the oil occasionally with a thermometer.

Make T_h 80° C. and keep it, as nearly as may be, at that temperature, stirring the oil frequently with a thermometer.

Make R such that the thermoelectric force of the copper-iron couple will produce an easily measurable deflection on the scale of M.

With these conditions read this deflection, D, and read the thermometers at the same instant. Then reverse at K, keeping the temperatures as nearly unchanged as practicable, and, as soon as the deflection is established, read it and the temperatures again. Continue in this way through several reversals at K, recording the readings in a table like that here indicated:

	T_l.	T_h.	Deflection.	
1st Case	......	......	......	
	......	......		......
	......	......	......	
	......	......		......
	......	......	......	
Means	......	......	......	......
		Mean D	......	

When this series of observations is completed, raise the temperature of the hotter oil to 140° C., or thereabout, so as to make the difference of temperature between the junctions about twice as great as before. Keep T_l as before.

Change R, if need be, so that the deflection now obtained shall not be too large for the scale of M.

With these conditions make a set of observations cor-

responding to the set indicated above. Call these the observations for the 2d Case.

Subscripts 1 and 2 will be used when it is necessary to distinguish between the 1st and the 2d Case.

For the thermo-electromotive forces developed in these two cases respectively we have:

$$E_1 = F \times D_1 \times (R_1 + R_m) \text{ volts,}$$

and

$$E_2 = F \times D_2 \times (R_2 + R_m) \text{ volts.}$$

If we assume, what may not be strictly true, that the two lines representing copper and iron on a thermo-electric diagram of the ordinary kind are straight lines, we can find the "neutral temperature" for a copper-iron couple by proceeding as follows, making use of the data just obtained: Lay off, as in Fig. 44, a horizontal line

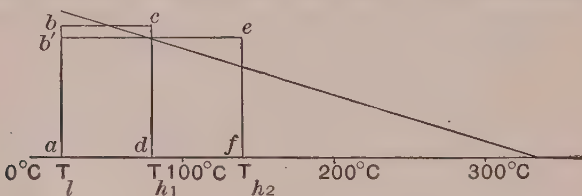


Fig. 44.

representing temperatures, and on this mark points corresponding to T_l, T_{h_1} and T_{h_2}. On $T_l \ldots T_{h_1}$ as a base erect a rectangle, *abcd*, representing, on any convenient scale, the electromotive force E_1 On $T_l \ldots T_{h_2}$ as a base erect a rectangle $ab'ef$, representing, on the same scale that was used for E_1; the electromotive force E_2. Through the midpoint of the line *bc* and the midpoint of the line $b'e$ draw a straight line, and continue this line till it crosses the temperature line. The point of crossing is the point of neutral temperature for the copper-iron couple studied. This point may be different for different copper-iron couples.

$Q = 1, \quad I = E$

$\Sigma I R = \Sigma E$

EXERCISE 26.

STUDY OF AN ELECTRIC MOTOR.

A direct-current, shunt-wound fan-motor of small size, one eighth horse-power, serves well for this Exercise; and such a motor will be assumed as the subject of study in what follows.

Disconnect the field coils from the armature and measure, by means of a Wheatstone bridge, the resistance, R_f, of the field coils.

Measure in the same way the resistance of the armature in several different positions, as this resistance will depend somewhat on the points of contact of the brushes with the commutator sectors, and take the mean of the several values thus found as R_a.

Reconnect field coils with armature and arrange apparatus according to the indications of Fig. 45, where A

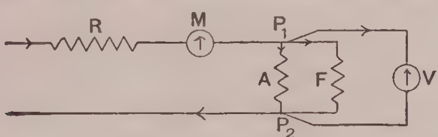


FIG. 45.

represents the armature; F represents the field coils; M represents an ammeter, reading the total current, i, directly in amperes; V represents a voltmeter, reading Δ,

the difference of potential between the poles of the motor, P_1 and P_2, directly in volts; R represents some resistance, capable of bearing without injury several amperes and variable at the will of the experimenter.

The current which drives the motor is obtained from some source not here represented.

Introduce such a value of R as will give the motor a very low speed, and while this speed is maintained, and is measured by means of a speed counter applied by hand to the end of the armature axis, take a reading of M and of V.

Make R such that the speed will be medium, and make measurements as before.

Make R such as to give the highest advisable speed, then measure as before.

The "efficiency" of the motor, which efficiency we shall call E, is defined as follows:

$$E = \frac{\text{Mechanical work done by motor per second}}{\text{Energy supplied to motor per second}}$$

$$= \frac{\text{Watts of output of mechanical work}}{\text{Watts of supply of energy}}.$$

The watts of supply are easily found for any one of the cases tried by multiplying i by Δ, it being assumed, as it fairly may be, that the very small current which goes through V spends there a negligible amount of energy.

The watts of mechanical output are obtained, approximately, by deducting from the watts of supply the watts spent in generating heat in overcoming the resistances, R_f and R_a respectively, in the field coils and the armature. If i_f is the current which runs through the field coils and i_a the current which runs through the armature, if H_f is the number of watts spent in generating heat in the field

coils, and H_a the number of watts spent in generating heat in the armature, we have

$$H_f = i_f \times R_f^2 \quad \text{and} \quad H_a = i_a \times R_a^2.$$

The current i_f can be found at once by Ohm's law. It is merely $\Delta \div R_f$; and H_f and be calculated accordingly.

The current i_a cannot be put down as $\Delta \div R_a$, for the reason that the rotation of the armature in the magnetic field generates what is called a counter electromotive force, which opposes and practically reduces Δ, so far as the armature is concerned. But we can find i_a very simply from the fact that $i = i_f + i_a$; and accordingly we can calculate H_a.

And so it is possible to calculate, approximately, the value of E for each of the cases tried.

(into h
watts g

$\Delta V \times I_m$

= diff. of pot. of mot
× current

EXERCISE 27.

STUDY OF A DYNAMO.

The dynamo to be studied is supposed to give a direct current of not many horse-power. It should be of simple construction and may be either shunt wound or series wound. It is supposed to be driven by the same steam engine that was studied in Exercise 16. The external circuit through which the dynamo drives a current should be so constructed as to carry the full output of the dynamo without injury and to allow of such variations in resistance as will vary the load of the dynamo from very light to heavy. The Exercise here planned does not require an exact measurement of this resistance.

In the external circuit which contains this resistance R (see Fig. 46), and leads from one pole, P_1, to the other, P_2,

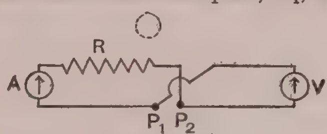


FIG. 46.

of the dynamo, insert an ammeter, A, to measure the external current, i_e. Between the same poles, but not in the same path with R, introduce a voltmeter, V, to measure Δ_e, the difference of potential available for sending the current through R.

Run the engine at such a number of revolutions per minute, N_s, as to give to the dynamo armature a suitable number of revolutions per minute, N_a, and keep this speed of the armature constant, as nearly as may be, during all the changes of load caused by changes of R. As there may be considerable slipping of belts between the engine and the dynamo when the work done by the latter is large, a speed counter should be applied directly to the axis of the armature.

Make and record observations according to the indications given in the table below, beginning with a small output of energy from the dynamo and going by as many as five or six stages to the largest output. In the column headed I is to be put, for each case, the mean area of the indicator diagrams taken from the two ends of the cylinder for that case.

Case.	i_ϵ.	Δ_ϵ.	N_a.	N_s.	I.
. . . . . .	. . . . . .	. . . . . .	. . . . . .	. . . . . .	. . . . . .
. . . . . .	. . . . . .	. . . . . .	. . . . . .	. . . . . .	. . . . . .
. . . . . .	. . . . . .	. . . . . .	. . . . . .	. . . . . .	. . . . . .

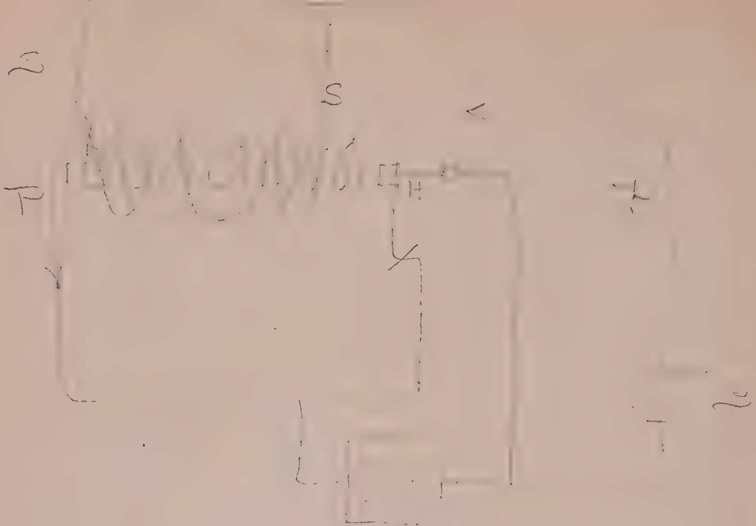
If there are some, but not large, variations in the N_a column, it will usually be allowable to make, from the data recorded in the table above, a revised table, in which all values of N_a are the same as any one actually observed value, and the columns headed i_ϵ, Δ_ϵ and N_s are obtained from the observed columns on the assumption that i_ϵ, Δ_ϵ, and N_s are proportional to N_a. The column headed I should be left unchanged.

From the values given in the columns headed i^ϵ, Δ_ϵ, and N_a in the revised table, plot on coordinate paper on a moderate scale the "characteristic" curve of the dynamo for the speed N_a.

Defining the efficiency of the engine-dynamo combination by the ratio

$$E = \frac{\text{Electrical horse-power of dynamo output}}{\text{Indicated horse-power of engine}},$$

find this efficiency for each of the cases observed, and with the values thus obtained plot, on the same paper with the "characteristic," an efficiency curve.


S
F
Condenser

EXERCISE 28.

EXPERIMENTS WITH AN INDUCTION-COIL, ETC.

Electric Current in Air at Various Pressures.

In Fig. 47, P_1 annd P_2 are the secondary poles of an induction coil capable of giving a 1.5 cm. spark in open air; T is a glass tube about 30 cm. long and 4 cm. in diameter, having a close fitting perforated rubber stopper at each end; O is a side tube for making connection with an exhaust pump; r_1 is a brass rod terminated by a brass ball about 1.5 cm. in diameter; t is a brass tube, closed at the outer end, in which slides a brass rod, r_2, similar to r_1 and terminated as r_1 is terminated. By inclining T and thus sliding r_2 the balls can be placed near each other or far apart.

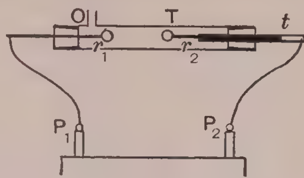


Fig. 47.

Place the balls so far apart that the induction-coil cannot send a spark between them in air at full pressure, and then, while the coil is working, pump out the air from T gradually till the lowest pressure easily attainable is reached. Note the character of the discharge between the balls at the various stages of exhaustion. When the

striæ appear, observe that the convexity of each luminous layer changes direction with change of direction of the primary current operating the induction-coil, etc.

Apply the induction-coil to various "Geissler" and "Crookes" tubes, particularly to that form of Crookes tube which shows a narrow streak of cathode rays, and show the deflection of this streak by action of a horse-shoe magnet.

With an induction-coil capable of giving a spark of not less than 10 cm. in open air, work a Roentgen-ray tube and with the use of a fluorescent screen examine the skeleton of the hand and wrist.

Electrical Resonance.

Lead a wire 2.5 m. long by a rectangular course from the bottom of the outer coating of a moderate-sized Leyden jar to a point about 1 cm. distant from the rod connected with the inner coating. Call this System 1. With an equal Leyden jar and an equal wire make a similar System 2; but here have the wire touch the rod leading from the inner coating, and lead a strip of metal from the inner coating to a point distant about 0.1 cm. from the outer coating. Place the two systems facing each other, 0.5 m. apart. Remove the Leyden jars of a Holtz or other similar machine, separate the poles 2 cm., connect one pole with the inner and one with the outer coating of the jar of Sy tem 1, work the machine and, when discharge occurs in System 1 look for a spark in the narrow gap of System 2.

With the small induction-coil try some form of Herz-wave or wireless telegraphic apparatus, using a coherer. Observe the absorption or transmission of the waves by a wire grating and their reflection from a flat sheet of tin.

EXERCISE 29.

MISCELLANEOUS EXPERIMENTS IN SOUND.

1. *Bell in vacuum.*—To prevent sound from passing out by way of solids, place the support of the bell on a soft pad of folded cloth on the plate of the air-pump.

2. *Reflection of sound.*—Place a watch at one focus of a pair of concave mirrors facing each other and the mouth of an ear-trumpet at the other focus.

3. *Vibrating string.*—Exhibit nodes and loops of a vibrating monochord by means of Λ-shaped pieces of paper, some white and some colored.

4. *Chladni's figures.*—Look well to the mounting of the plates. Have several of the most easily obtained figures traced in pencil on the plates as a guide to the fingers and the bow.

5. *Pair of tuning-forks.*—Take two mounted tuning-forks of nearly or quite the same pitch and, by loading both prongs of one of them with equal lumps of wax, make them when sounding together give first many, then few, then no beats a second. When the last condition is reached, try for "sympathetic" vibrations.

6. *Organ-pipes.*—Examine various forms, noting peculiarities of general shape, of *embouchure*, etc.

7. *The siren.*—In finding the pitch of an organ-pipe by means of the siren, a person who has not a good musical ear will do well to give attention to the beats produced between the note of the siren and the note of the pipe, maintaining approximate unison by regulating the pressure of the air from the bellows, or by so touching the revolving spindle of the siren that these beats shall be infrequent. Indeed, this means of testing unison is more efficient than the mere sense of harmony in the most accurate ear.

8. *Lissajou's curves.*—Use, if they are available, electrically driven tuning-forks, each carrying a small mirror. The source of light should be a small hole, not more than 0.1 cm. in diameter, illuminated from behind by a powerful lamp.

9. *Limit of audibility.*—Use tuning-forks or rods of exceedingly high pitch and find the highest note which any one of several persons can hear when the ear is placed close to the sounding body.

Note: It is well to make a plan of the arrangement of laboratory tables and mark on this plan the best position for each piece or set of apparatus which is to be used year after year.

EXERCISE 30.

THE SEXTANT, ETC.

Measurement of Angle of Elevation.

In Fig. 48, M_1 is the movable, pivoted mirror of the sextant; M is the fixed mirror, one lateral half silvered,

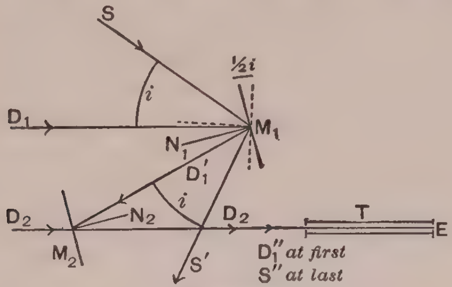


Fig. 48.

the other lateral half clear; T is the telescope or, better for the beginner's use, a mere sighting-tube; N_1 is a perpendicular to M_1, and N_2 is a perpendicular to M_2; S is a ray of light coming from some well-marked point or horizontal line on some object a hundred meters or more distant; D_1 is a ray coming from some well-marked distant horizontal line; D_2 is another ray coming from the same horizontal line from which D_1 comes and passing through the unsilvered part of M_2 directly to the eye at E.

The object of our use of the sextant is to measure the angle i between S and D_1. For this purpose two operations are necessary:

First: M_1 is set, as in Fig. 48, so that D_1', the once reflected D_1, falls on M_2 in such a way that D_1'', the twice reflected D_1, will go parallel to and side by side with D_2 to the eye. When this condition is reached, the two parts of the lower horizontal line from which D_1 and D_2 come, seen by way of the unsilvered and the silvered parts of M_2 respectively, will appear continuous with each other. The first reading of the indicator on the arc of the sextant should then be made, with use of the vernier, and recorded. Call this reading α_1.

Second: Turn M_1, clockwise in Fig. 48, until the ray S', which is S once reflected, takes the direction M_1M_2, and S'', which is S twice reflected, takes the course M_2E. Then take another reading on the arc. Call this α_2.

In going from the first position to the second, M_1 has been turned through an angle equal to $i \div 2$. The arc, however, is graduated in such a way that the difference, $\alpha_1 - \alpha_2$, of the readings made on it is equal to the angle i.

Note: If the upper and lower lines or points looked at are not some hundreds of feet distant from the observer, a little change of his position forward or backward will affect the angle i perceptibly.

Camera Lucida.

Examine and try some simple form of this instrument.

EXERCISE 31.

COMBINED LENSES.

FORMULA FOR TWO LENSES COMBINED.

O in Fig. 49 is a point sending rays directly to the lens A, and I is the point to which the action of A, alone,

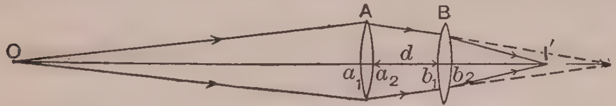


FIG. 49.

would concentrate the rays. I' is the point at which they actually are concentrated by the combined action of A and B. The distance a_2b_1 between the lenses is called d.

Let F_a = focal length of A and F_b = focal length of B;
" a_1O be called D_o and a_2I be called D_i;
" b_1I " " $D_{o'}$ " b_2I' " " $D_{i'}$.

For A we have

$$\frac{1}{D_o}+\frac{1}{D_i}=\frac{1}{F_a}, \quad \text{whence} \quad D_i=\frac{D_o \times F_a}{D_o-F_a}. \quad . \quad . \quad (1)$$

For B we have

$$\frac{1}{D_{i'}}-\frac{1}{D_{o'}}=\frac{1}{F_b}, \quad \text{whence} \quad \frac{1}{D_{i'}}=\frac{1}{F_b}+\frac{1}{D_i-d}. \quad . \quad (2)$$

Substituting for D_i from equation (1) we get

$$\frac{1}{D_{i'}}=\frac{1}{F_b}+\frac{D_o-F_a}{D_oF_a-d(D_o-F_a)}. \quad . \quad . \quad . \qquad (3)$$

When D_o is very large compared with F_a, we may write

$$\frac{1}{D_{i'}}=\frac{1}{F_b}+\frac{D_o}{(F_a-d)D_o}=\frac{1}{F_b}+\frac{1}{F_a-d}. \quad . \quad . \quad . \qquad (4)$$

If B were a diverging lens of focal length F_b, the last formula would be

$$\frac{1}{D_{i'}}=\frac{1}{F_a-d}-\frac{1}{F_b}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

Experimental Study of Two Converging Lenses in Combination.

The formula (4) derived above is here to be tested in a number of simple cases.

In Fig. 50, T is a wooden trough about 30 cm. long, open at the ends, see Fig. 51, and having a sort of cushion or padding along its inner vertical surfaces. It is so pro-

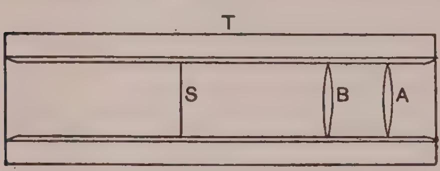


Fig. 50.


Fig. 51.

portioned and constructed as to be a convenient holder of the lenses A and B, each of which is double convex, thin, and of a focal length not less than 15 cm. nor more than 25 cm. S is a white movable screen.

Put A and S alone in T, open a window and, going well back from this window, so that S will get but little direct light from the sky, place T so that the light from some distant object out of doors will pass through A and fall on S. Move either A or S back and forth, keeping each squarely across T, until the condition is found which makes the image of the distant object as distinct as it can be made on S; then measure the distance from S to the nearer face of A, and call this F_a.

In the same way find F_b, the focal length of B.

Then use both lenses at once, as in Fig. 50, making d, the distance between them, at first 0 cm., then 2 cm., then 4 cm., then 10 cm., and measure in each case $D_{i'}$, the distance from S to the nearer surface of B when the image of the distant object is most distant.

From the formula (4),

$$\frac{1}{D_{i'}}=\frac{1}{F_b}+\frac{1}{F_a-d},$$

calculate a value of D for each of the four cases used, and compare for each case the calculated value with that found by experiment.

Study of a Converging and a Diverging Lens in Combination.

Extend or vary this Exercise by use of a combination in which B is a concave, or diverging, lens of focal length considerably greater than F_a.

To find the focal length of such a lens proceed as follows: Place between the lens and some distant object an upright wire or pin, P in Fig. 52, and then, keeping the eye some

30 cm. distant from the lens, B, look over the lens at P and through the lens at I, the image of the distant object, and vary the distance BP till P seen over the lens and I seen through the lens, simultaneously, have no parallax, that is,

FIG. 52.

do not separate laterally from each other when the eye is moved to the right or left.

The formula to be used for this case is the one already given as (5),

$$\frac{1}{D_{i'}} = \frac{1}{F_a - d} - \frac{1}{F_b}.$$

EXERCISE 32.

TELESCOPE AND MICROSCOPE.

KIND, NUMBER AND ARRANGEMENT OF LENSES IN A TELESCOPE.

The telescope to be studied is one of the familiar, inexpensive instruments, constructed according to the general plan shown in Fig. 53, which show an object right-side up.

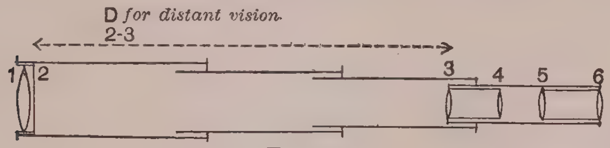


FIG. 53.

Unscrew the socket which holds the lenses 1 and 2, which together make the objective, and then, holding this socket in such a way that the lenses cannot spill when it is opened, unscrew the retaining ring which secures the lenses in place. In removing these from the socket note carefully the way in which they are placed with respect to each other, so as to be able to replace them without error.

Measure F_1, the focal length of 1 alone.
" F_2, " " " " 2 "
" $F_{1.2}$ " " " " 1 and 2 together.

Replace 1 and 2, after wiping them clean with a soft cloth or a piece of chamois skin, and focus the telescope on some distant object. Contrive in some way to find the distance, $D_{2\cdot 3}$, which lies between lens 2 and lens 3, when the telescope is in this condition.

Draw out from the inclosing tube the tube which holds the combination 3–4–5–6, and see whether this combination, with 6 held close to the eye, will act as a positive eyepiece, that is, give a distinct image of a small object a little beyond 3. If it will do so, observe how far this small object must be from 3 in order to be distinctly seen, and call this distance D_3. Observe also whether the image is upright or inverted as compared with the object.

Make a diagram of the telescope, similar to Fig. 53, on a scale of one half, the three distances $F_{1.2}$, $D_{2.3}$ and D_3.

Magnifying Power of Telescope.

Mount the telescope in a horizontal position on some convenient support and direct it at a vertical rod, at least five or six meters distant, on which are alternate white and black horizontal stripes each 5 cm. wide. Adjust the telescope until it gives distinct vision of the stripes on the rod. Look with one eye through the telescope at this image and look at the same time with the other eye alongside the telescope directly at the rod. By partly closing, or otherwise partly obstructing the vision of, one eye, contrive to make the two eyes give simultaneous independent views of the stripes on the rod, producing an effect resembling that shown in Fig. 54, one stripe seen through the telescope extending over a considerable number of stripes seen by the unassisted eye. This number is the magnifying power of the telescope as here used.

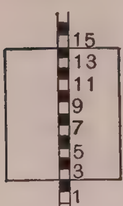


Fig. 54.

Microscope.

Examine the construction and mode of operation of a working compound microscope of simple form.

EXERCISE 33.

INDEX OF REFRACTION.

INDEX OF REFRACTION OF A PRISM.

A SIMPLE spectrometer is to be used for this purpose, one having a main circle about 15 cm. in diameter and reading, let us say, to 1′.

Adjustment of spectrometer.—Remove the prism, P, in Fig. 55, direct the telescope T toward a luminous surface and adjust the eyepiece so as to give a distinct view of the cross-hairs indicated by the +. Next point T toward some distant object and so focus it, by pushing in or pulling out the tube containing the eye-lens and the cross-hairs, as to get a distinct view of that object, with the cross-hairs apparently fastened rigidly on it. The telescope is then said to be focused, or adjusted, for parallel rays.

Bring T thus adjusted into line with the collimator, C, and so adjust C, by pulling out or pushing in the tube which bears the slit S, that an observer looking through T perceives a distinct image of S, which should be well illuminated during this operation. Make S as narrow as

it can well be without causing the image of it to look discontinuous.

Measurement of refracting angle of prism.—Place the prism P on the central turntable, or small circle, g, as in Fig. 55. See that the slit S is vertical and well illuminated. Place T in a position with respect to C like that

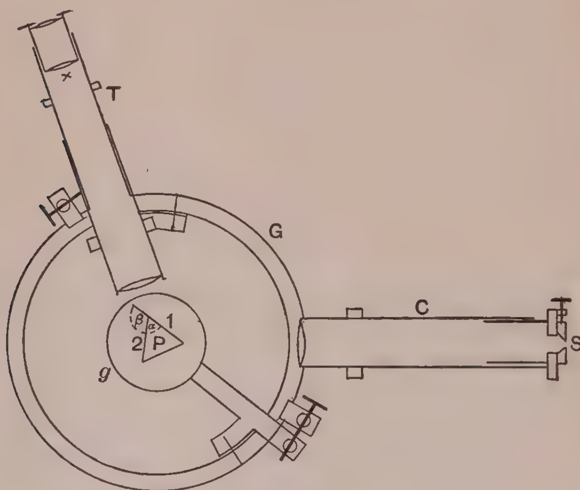


FIG. 55.

shown in the figure and fasten both in position. Turn the table g until the face 1 of P is in such a position that light falling upon it from S through the lens of C is reflected into T and forms an image of S on the intersection of the cross-hairs seen in T. Then read θ_1, the angle which shows the position of g.

Keeping T and C unmoved, turn g, taking care now not to let P slip on g, clockwise until the face 2 of P comes into such a position as to do, for the light coming from S, just what the face 1 did before. Face 2 will now be

parallel to the original direction of face 1, and the prism will have been turned through the angle β. Read θ_2, the angle showing the present position of g. From θ_1 and θ_2 can be found β. The angle α, which is the angle wanted for our purpose, is found by subtracting β from 180°.

Measurement of minimum deviation of light passing through faces 2 and 1 of the prism.—Illuminate s with sodium flame, F, Fig. 56, that is a Bunsen flame in which

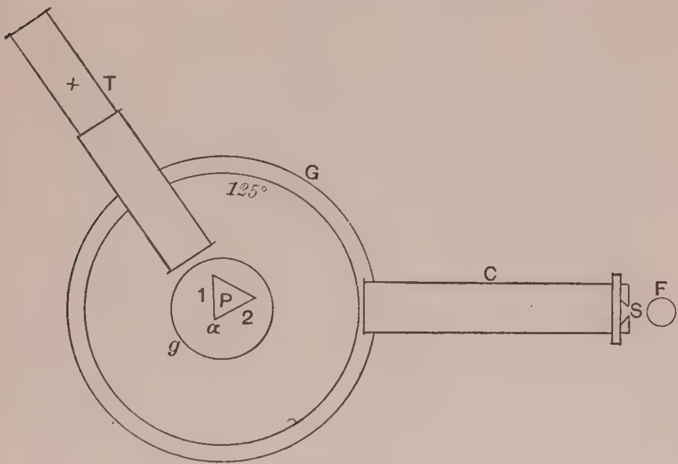


Fig. 56.

is held a lump or bead of common salt. Remove P and set T in line with C, which remains fastened, bringing the image of S exactly on the cross-hairs of T, and then take a reading, γ_1, which shows the position of T on the large circle G. Next place T, with respect to C, as in Fig. 56, making the angle between the two about 125°, replace P on g and turn g into such a position that light coming from S will pass in at face 2, out at face 1, and enter T in such a way as to give an image of S therein.

While looking through T, try whether it is possible by turning g to make this image of S move out of the field of view toward the right. If this is not possible, turn g so as to carry the image as far to the right as it will go, fasten g, and then turn T until the intersection of the cross-hairs falls on the image of S. Then read γ_2, the angle showing the present position of T on G. From γ_1 and γ_2 can be found δ, the angle of minimum deviation of sodium light passing through P.

If with the first position of T, as shown in Fig. 56, it is possible to make the image of S go out of the field toward the right, move T somewhat to the right, following up the image which has disappeared, until T is in such a position that the image of S cannot go out of the field of view toward the right. Then follow the directions given in the preceding paragraph.

The index of refraction, n, is found from α and δ by means of the formula

$$n = \frac{\sin\left(\frac{\alpha + \delta}{2}\right)}{\sin\frac{\alpha}{2}}.$$

Apparent Depth of Water.

It is proved in text-books that the apparent depth of an object in water, for an eye looking vertically down at the object from the air above, bears to the real depth the ratio $\frac{1}{n}$, where n is the index of refraction of water. It should therefore be possible by measuring the apparent depth in a given case and comparing this with the real depth to get the value of n.

In Fig. 57, V is a tall vessel nearly filled with clear water; T is a telescope capable of being focused on a near object; C is some well-defined flat article, a copper coin for example. T should be held in some firm support which will keep the objective at a fixed height.

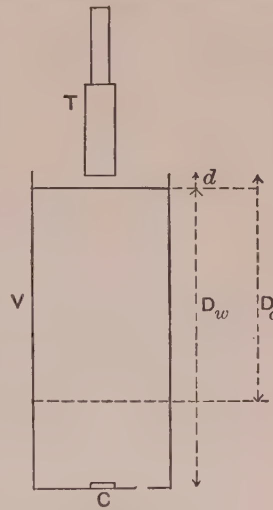


FIG. 57.

Direct T toward C and focus it so that C will be distinctly seen through it. Measure D_w, the depth of the water over C, and d, the air distance between the water and the objective of T. Then remove the jar and, without changing T in any way, find the distance, D_a, from the objective to some object held in air at such a distance beneath as to be distinctly seen through the telescope.

Then we have

$$D_w \text{ of water} + d \text{ of air} = D_a \text{ of air},$$

or

$$D_w \text{ of water} = (D_a - d) \text{ of air};$$

whence

$$n = \frac{D_w}{D_a - d}.$$

EXERCISE 34.

INTERFERENCE OF LIGHT-WAVES.

MEASUREMENT OF WAVE-LENGTH WITH A BIPRISM.

Theory.—A and B in Fig. 58 are supposed to be perfectly equal and similar small sources of homogeneous light, each sending out toward the right simultaneous as well as equal radiant impulses, all of one wave-length. C is a point half-way between A and B. M is a point lying

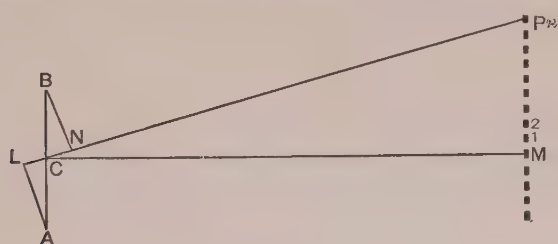


FIG. 58.

on a line drawn perpendicular to $\overline{AB}$ from C, and the distance $\overline{CM}$ is supposed very large compared with the distance $\overline{AB}$. At M the impulses from A and B meet in the same phase and make a bright spot, part of a bright band extending perpendicular to the plane of the figure. At 1, distant one wave-length, λ, farther from A than from B, is another bright spot; at 2, distant 2λ farther from A than

from B, is still another; and so on; P_n is the location of the nth bright band from M. The distance $\overline{P_nM}$ is very small compared with $\overline{CM}$. $\overline{BN}$ is drawn perpendicular to the line $\overline{P_nC}$; $\overline{AL}$ perpendicular to $\overline{P_nC}$ extended.

The triangles CBN, CAL, and PCM are all similar. Hence

$$\left.\begin{array}{l}\overline{CB}:\overline{CN}::\overline{PC}:\overline{PM},\\ 2\overline{CB}:\overline{LN}::\overline{PC}:\overline{PM},\end{array}\right\}\text{ strictly;}$$

$$\left.\begin{array}{l}\overline{AB}:[n\lambda]::[\overline{MC}]:\overline{PM},\\ \lambda=\dfrac{\overline{AB}\times\overline{PM}}{n\times\overline{MC}},\end{array}\right\}\text{approximately.*}$$

The biprism, see P in Fig. 59, furnishes the two sources needed, by so acting on the light received from the illumi-

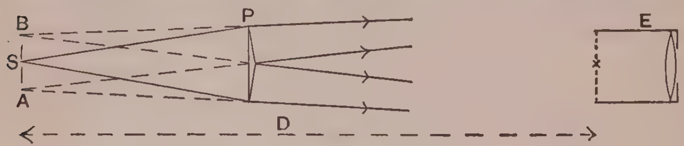


FIG. 59.

nated slit S as to give two equal virtual images of S. at B and A respectively.

Experiment.—Mount S, P, and the eyepiece, E, Fig. 59, all at the same level on the "optical bench". The distance from S to P may well be 40 cm. D, the distance from the plane of S to the plane of the cross-hairs of the eyepiece, corresponds to the distance $\overline{CM}$ in Fig. 58. It may well be 100 cm.

Take great care to have S and P vertical, so that they shall be parallel to each other, and placed squarely across

* Strictly, $n\lambda=\overline{PA}-\overline{PB}=(\overline{PL}+)-(\overline{PN}+)=\overline{LN}$ approximately.

the bench; otherwise, the bands seen in the field of the eyepiece will be unsatisfactory.

Illuminate S by a strong sodium flame, and place a shield in such a position that no light from this flame shall reach E except through S. Work finally in a darkened room. Use that width of S which on trial proves to give the most distinct and measurable bands in the field of the eyepiece.

In measuring the distance which corresponds to $\overline{PM}$ in the formula derived above it is not necessary to begin at the middle band. As the number of distinct bands to be seen, under the circumstances of this Exercise, is few at the best, it is well to proceed in this matter as follows: By means of the screw which gives lateral movement to E set the cross-hairs, indicated in Fig. 59 by ×, on the extreme left-hand band which is distinct, and then take a reading on the scale belonging to the screw. Next turn the screw in such a way as to move the cross-hairs over to the extreme right-hand distinct band, and take another reading* on the scale. Count n the number of intervals from the first to the last band used.

Of the quantities needed for the determination of λ, by the formula given above, we have now obtained all except the distance $\overline{AB}$. To find this distance we can use the

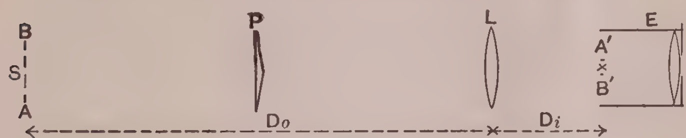


FIG. 60.

following method: Keeping S, P and E at the same distances as before, mount a lens L, about 15 cm. in focal length,

* To avoid error from "back-lash," or looseness of the screw in its socket, the approach to any setting in which a reading is to be taken should always be made in the same direction.

on the optical bench between P and E, see Fig. 60, and move it back and forth until it is in such a position as to give in the field of view of E distinct images, A' and B', of A and B. Measure the distance $\overline{A'B'}$. Measure also D_o and D_i. Then, by the ordinary rule of magnifying power,

$$\overline{AB} = \overline{A'B'} \times \frac{D_0}{D}.$$

Newton's Rings.

Examine some form of apparatus producing these interference effects.

Diffraction Rings.

Look through a pin-hole in a card at the image of the sun in a small bead of mercury, and note the rings of light seen surrounding the image.

Effect of Small Apertures * for Light waves.

Look through the meshes of a very fine wire screen, held close to the eye, or through fine pin-holes in a card, at a printed page, and note the obscuration of the print.

* See Hastings and Beach's *General Physics*, § 584.

EXERCISE 35.

PHENOMENA OF POLARIZED LIGHT.

Polarization by Reflection.

Study the phenomena presented by the polariscope of Malus, two unsilvered plates of glass mounted one over the other on the same support:

First.—Set the two reflecting surfaces parallel to and facing each other, each making an angle of 33° with the line joining their centres, which line we will call the axis of the system. Place some well-lighted object in such a position that the first mirror will reflect light from it to the second mirror and the latter will send it to the eye of the observer, kept near the second mirror. Watch the image of this object in the second mirror while turning this mirror about the axis of the system. With what position of the rotating mirror is the image most distinct? With what position is it least distinct?

Second.—Vary considerably the angle which both mirrors make with the axis of the system, keeping the mirrors parallel, and then try once more the effect of rotating the second mirror.

Polarization by Tourmaline.

Try the ordinary experiment with plates of tourmaline in the "tongs".

Optic Axis of Calc Spar.

Use a crystal of calc spar having two of its faces so cut that the line joining the centres of these two faces is perpendicular to each and is parallel to the optic axis of the crystal. Rest the crystal on one of these faces on a printed sheet and look down through the other face at the print, noting that when, and only when, the line of sight is perpendicular to the faces, each letter is seen single.

Nicol Prism.

Examine a Nicol prism, and use it as an analyzer in examining the light polarized by reflection from glass or by transmission through tourmaline.

Color Effects; Elliptic Polarization.

Study the color effects produced by the transmission of plane-polarized light through plates of selenite or of mica, using the Norremberg apparatus.

Double Refraction in Strained Glass.

Examine the state of unannealed or temporarily strained glass by sending plane-polarized light through it and receiving the transmitted beam through a Nicol prism.

Rotation of Plane of Polarization by Quartz.

Use homogeneous light, for example the sodium flame, with a polarizer and an analyzer, setting the latter to extinction of the light received by it. Then introduce between the two a plate of quartz, cut perpendicular to its optic axis. After observing that the analyzer no longer extinguishes the light completely, rotate it on its axis, and see whether in any position of rotation it will again extinguish the light.

MAGNETIC ROTATION OF THE PLANE OF POLARIZATION.

Use a powerful electromagnet having a straight bore extending through the iron core and the poles, with a polarizer at one end of this bore and an analyzer at the other end. Have the poles about 5 cm. apart, so as to admit between them a bar of "heavy glass" about 4.5 cm. long. Use a sodium flame in front of the polarizer and, the glass bar being in place but the electric current not being turned on, set the analyzer to extinction. Then, while looking through the analyzer, turn on the current. With the current on set the analyzer once more to extinction; then reverse the current and note the effect. Find how much the plane of polarization is rotated by reversing the current.

APPENDIX.

Torricelli's Law.

In deducing Torricelli's law for the velocity of flow of a liquid from an orifice, we assume that friction plays no part in the action. We assume that the kinetic energy of forward motion of any amount of the liquid just after its escape is equal to the difference between its potential energy just before and just after escape. This difference of potential energy is equal to the work done on the liquid from behind by the expelling forces minus the work done by the liquid in overcoming the opposing forces in front during its expulsion.

This net amount of work done on a given volume of liquid during outflow is just equal to the amount which would be done on an equal volume of matter expelled in solid form from the same interior place to the same exterior place; for, if this were not true, we could, by forcing liquid into a reservoir while enclosed in thin solid envelopes and letting it out from the reservoir in its free fluid condition, or *vice versa*, gain work from the whole operation and thus cheat the law of conservation of energy.

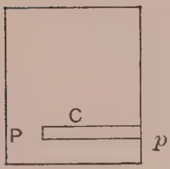


FIG. 61.

In Fig. 61, let C be a rigid column of length L cm. and of cross-section s sq. cm. in a place where the pressure is P grams per sq. cm. Let this column be forced out through a hole just large enough to permit its passage into a place where the pressure is p grams per sq. cm. The total expelling force is Ps grams; the total opposing force ps grams. The kinetic energy of C after its expulsion, equal to the net work of expulsion, is, in gram-centimeters,

$$K.E. = Ps \times L - ps \times L = (P-p)Ls = (P-p)V,$$

where V is the volume expelled. Thus, $K.E.$ is a quantity independent of the shape of C and of the size of the orifice.

If we consider 1 gram of a liquid of density D, the volume will be $\frac{1}{D}$, and we have, v being velocity after expulsion,

$$K.E., \textit{ in gm.-cm.}, = \frac{v^2}{2g} = \frac{P-p}{D},$$

or

$$v = \sqrt{2g \cdot \frac{P-p}{D}}.$$

The difference of pressure, $P-p$, may be maintained in any one of various ways. If it is due merely to the weight of liquid above the level of the orifice, as when water flows from the side of an open-topped reservoir into the air, the depth of liquid, above the orifice, needed to make this difference of pressure is $(P-p) \div D$. If we call this depth, or height, or "head," h, our formula for v is

$$v = \sqrt{2gh},$$

which is Torricelli's law.

If we undertake to apply the preceding course of reasoning to the flow of a gas, we are met by the difficulty that D is for a gas not a constant. This makes various complications. The difference of potential energy between the first and last states of the gas is not simply $(P-p)V$; for the volume, V, is not the same at the start as at the end. Moreover, a gas may during expansion gain kinetic energy at the expense of its own heat energy. If, however, the change of pressure, $P-p$, is small compared with P, we can use as approximately true for a gas the formulas derived above for a liquid; and in this case h means the height of a column of the gas, supposed everywhere of the density D, which it has just before outlet, which would be required to produce the difference of pressure $P-p$.

Fluid Friction.

Although friction is ignored in the derivation of Torricelli's law, it really plays a very important part when rapid relative motion is maintained between a fluid and a solid, as when a ship is driven at a high speed or some fluid is forced swiftly through a long, narrow tube. The law of dependence of this friction on the magnitude of the relative motion of the solid and fluid is not simple; but within a considerable range of this velocity the frictional resistance is approximately proportional to its square. Friction is approximately proportional also to the area of the surface of slip between solid and fluid. Accordingly, we may write for R, the force, or resistance, of friction along a surface of extent S between a solid and a fluid, the relative velocity of the two being v,

$$R=kSv^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where k is a constant quantity in the sense that it is inde-

pendent of S and of v, though it may vary with the nature of the solid surface.

There is a certain advantage in writing instead of the constant k the constant $(fD \div 2g)$, in which D is the density of the fluid, assumed to be constant, g is the acceleration of gravity, and f is a constant quantity of such magnitude as to make $(fD \div 2g) = k$. With this change our formula (1) becomes

$$R = fDS\frac{v^2}{2g}. \qquad (2)$$

The factor f is called the *coefficient of friction.* For water, if R is expressed in gravitation units, the value of f may vary from 0.004, for a clean, varnished surface, to 0.009, for a surface like that of medium sandpaper. We shall, following the authority of the Scotch engineer Rankine, take $f = 0.0075$ in the case of water.

The height H, from which a body must fall freely to acquire a velocity v, being $= v^2 \div 2g$, we may substitute this H for $v^2 \div 2g$ in formula (2) and thus get

$$R = f(DSH). \qquad (3)$$

This expression for R is interesting; for, as S is a surface, H a height, and D the density of the liquid, it is evident that R, the resistance of friction, is equal to f times the weight of a column of the liquid having a base equal to the frictional surface and a height equal to that through which a body must fall in order to acquire a velocity equal to the velocity of slip.

If now we consider the amount of resistance due to friction of a fluid in a tube of length l and radius r, we have

$$R = fDS\frac{v^2}{2g} = fD(l \times 2\pi r)\frac{v^2}{2g}. \qquad (4)$$

An equal resistance might be offered by a resisting vertical column of the liquid. Thus Fig. 62 may represent

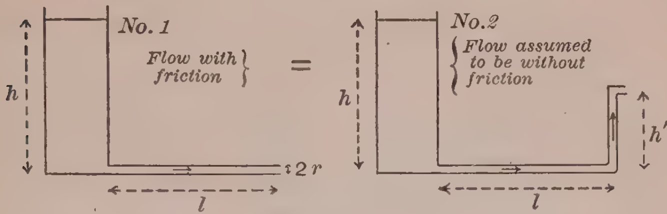


FIG. 62.

two cases of flow exactly similar, except that the friction in the pipe in the left-hand diagram is exactly replaced in retarding effect by the opposing column, of height h', in the right-hand diagram. To find the necessary height, h', we write

Wt. of water in vertical column = R of first case,

or

$$h' \times \pi r^2 \times D = fD(l \times 2\pi r)\frac{v^2}{2g},$$

or

$$h' = f\frac{l}{r}\frac{v^2}{g}. \quad \dots \dots \quad (5)$$

The height h' is called the "loss of head" caused by friction in case No. 1.

The velocity of outflow from No. 2, which is the same as the velocity of outflow from No. 1, is, by Torricelli's law,

$$v = \sqrt{2g(h-h')}; \quad \dots \dots \quad (6)$$

and, substituting for h' from (5), we get

$$v = \sqrt{2g\left(h - f\frac{l}{r}\frac{v^2}{g}\right)}. \quad \dots \dots \quad (7)$$

Solving this equation for v, and writing d in place of $2r$, we get

$$v=\sqrt{2gh \div \left(1+4f\frac{l}{d}\right)}. \quad . \quad . \quad . \quad . \quad (8)$$

Even for the flow of a gas through a tube, when the circumstances are such that the change of density of the gas may be ignored, that is, when the change of pressure and temperature during flow is small formula (8) may be used as approximately correct, with the same value of f which holds for water.